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Two methods for predicting the reliability and maintainability (R/M) of systems are discussed--a simulation method and an analytic method. Two computer programs (SIM3 and GEMJR) incorporating these methods and their input and output are described. The simulation method uses Monte Carlo techniques in predicting reliability. The analytic method incorporates the Poisson

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failure process to develop stochastic matrices which can be solved using infinite series to give reliability and availability.

The advantages and disadvantages of both methods are discussed. System configuration changes and complex missions can be considered more effectively using the simulation method. However, the simulation method does not calculate availability and provides only approximate results. In contrast, the analytic method predicts exact results and can examine such maintenance aspects as repairmen, standbys, and redundancies. Both methods are useful tools depending upon the R/M applications.

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ABSTRACT

Two methods for predicting the reliability and maintainability (R/M) of systems are discussed--a simulation method and an analytic method. Two computer programs (SIM3 and GEMJR) incorporating these methods and their input and output are described. The simulation method uses Monte Carlo techniques in predicting reliability. The analytic method incorporates the Poisson failure process to develop stochastic matrices which can be solved using infinite series to give reliability and availability.

The advantages and disadvantages of both methods are discussed. System configuration changes and complex missions can be considered more effectively using the simulation method. However, the simulation method does not calculate availability and provides only approximate results. In contrast, the analytic method predicts exact results and can examine such maintenance aspects as repairmen, standbys, and redundancies. Both methods are useful tools depending upon the R/M applications.

SECTION 1

INTRODUCTION

A January 1971 Navy instruction states, "It is the policy of the Department of Navy that logistic support planning will be included in the design, test, evaluation, production and operation of systems/equipment at all stages beginning with early conceptual studies."[1*] Logistics support planning has many facets. One of those delegated to the Chief of Naval Material involves the development and promulgation of "techniques for predicting costs and optimizing life cycle logistic support through analysis of potential tradeoffs between reliability, maintainability, design and manning interfaces, and other logistic support alternatives."[1] Consequently, the ability to determine reliability and maintainability (R/M) of systems throughout the design phase is necessary.

The theory for calculating R/M has been in existence for years but had not been widely applied before the advent of the digital computer. Evaluation of the performance of Navy systems--including ship systems--during the design phase has been facilitated through the application of R/M computer programs. This report describes two computer programs utilizing two different methods for calculating R/M: GEMJR and SIM3. GEMJR utilizes an analytical method; SIM3 a simulation method. GEMJR is based on GEM,[2] a large, comprehensive, user-oriented computer program which can solve many different R/M problems. Much smaller than GEM, GEMJR is useful for specific solutions; however, a user can easily develop a program suitable for his particular problem by following the analytic method incorporated in GEMJR. The simulation program SIM3[3] uses Monte Carlo[4] techniques for generating failure and repair events. From the descriptions of these two programs provided in this report, the user will be able to select the one best suited to his needs.

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*A complete listing of references is given on page 105.

SECTION 2

BACKGROUND

Reliability is defined as the probability that a system will perform satisfactorily--that is, without failing--for a given period of time.[5] The reliability of a system is an important consideration in logistics planning, for the fewer the failures, the less the maintenance required. However, the process for increasing system reliability must be considered in conjunction with the life cycle cost; increasing system reliability may increase system cost even though it may decrease maintenance cost. The tradeoffs between cost and reliability are beyond the scope of this report. (Appendix A is an example of a tradeoff analysis.) Only the methods used to determine reliability are described here.

Maintainability,[5] defined as the capacity of a system to be restored to operable condition within a given length of time after the system fails, must also be considered in logistics planning. Maintainability is not calculated directly; it is determined from availability, the probability that the system will be available for use at a given time. When a system fails, the length of time the system is inoperable is affected by the maintenance resources (number of repairmen, spare parts, etc.) available. Tradeoffs between maintenance resources and cost and time to repair are part of the logistics planning process.

System downtime (the time the system is inoperable due to failure) is affected by the system design as well as by the maintenance resources available. For example, standby equipment can be incorporated into the system to be used when the on-line equipment fails; redundant circuits can be included in the system for use when the primary circuit fails. Such design options affect the cost of a system and form a logical part of the logistic planning process.

SECTION 3

SIMULATION RELIABILITY PREDICTION USING SIM3

The SIM3 simulation method uses Monte Carlo techniques[6] in predicting reliability. Although it does not predict availability and gives only approximate results, it does provide useful information in R/M applications. For a more detailed description of the SIM3 computer program, see Appendix B.

3.1 SIM3 INPUT

The input to SIM3 is of three types:

- Descriptions of the phases of the mission scenario
- A system definition in the form of a reliability block diagram for each subsystem involved in each phase of the mission
- Reliability data in the form of mean time between failures (MTBF), mean time to repair (MTTR), and utilization factors (the average percentage of time the equipment is used during the mission)

3.1.1 Mission Scenario

A simple mission for a ship system may consist of a single phase--for example, constant speed over a given time. For such a specification the system definition and thus the configuration of its subsystems remains unchanged over its operational profile, referred to as the mission scenario. A complex mission will involve several phases, and the ship equipment configuration may change with each phase to perform the required operations. Addition or substitution of equipment or of whole subsystems may be required. For instance, one phase might call for half power, requiring only one boiler subsystem in the configuration. A later phase might need full power and the second boiler subsystem would be required. SIM3 can easily accommodate such equipment configuration changes.

3.1.2 System Definition

A system is a combination of equipment, components, and parts which perform the overall functions dictated by the mission. A complex system

can be divided into one or more subsystems, each of which performs a specific function in the system. Thus, a system is essentially an integrated collection of subsystems, and a system configuration incorporates all the individual subsystem configurations.

A system is defined for R/M purposes by a *reliability block diagram* showing the equipment in the system arranged to enable the calculation of R/M characteristics (see Appendix C). When the mission scenario specifies changes in performance from phase to phase, the equipment configuration, and thus the system definition, may change correspondingly.

3.1.3 Reliability Data

The reliability of a system is derived as a composite function of the probability distributions for each piece of equipment in that system. The probability distribution for each equipment is based on its operating history. Once the most appropriate distribution has been determined through statistical means, the parameters required in the distribution can be determined.

SIM3 uses an exponential distribution which requires the parameters mean time between failures (MTBF) and mean time to repair (MTTR) for each piece of equipment in the system. These parameters are determined by observing the occurrence of failures and times to repair of the equipment under actual operating conditions. By averaging the failure and repair data over a given time interval, mean values are obtained. Although an exponential distribution is not always the best approximation of mechanical system operations, it is used because the parameters MTBF and MTTR are more easily obtained than the parameters for most other distributions.

3.2 SIMULATION DESCRIPTION

3.2.1 Definitions

The use of simulation to determine the reliability of a system assumes knowledge of the operation of the system in its environment. The system's operation is simulated by generating failures and repairs of all the

equipment in the system over a specified mission scenario. By the use of appropriate criteria, the success or failure of the simulated mission can be determined.

Because the generation of failures and repairs in the simulation is random, the success or failure of a single mission is not meaningful. Many simulated missions must be run to obtain the reliability of the system, which is defined as the ratio of successful missions to total missions simulated.

Two examples are given to illustrate the application of Monte Carlo techniques to generate failures and repairs in a simulated mission. Reliability with repair is to be computed over a specific mission time period. To perform the simulation, the quantities TTF (time to failure) and TTR (time to repair) are generated for each equipment. TTF represents an interval from the time the equipment begins operation to the time when it fails. TTR represents the length of time required to repair the equipment after it has failed. The following algorithms are used to obtain TTF and TTR for an equipment with mean time between failure of MTBF and mean time to repair of MTTR:

$$TTF = -MTBF \ln RN$$

$$TTR = -MTTR \ln RN$$

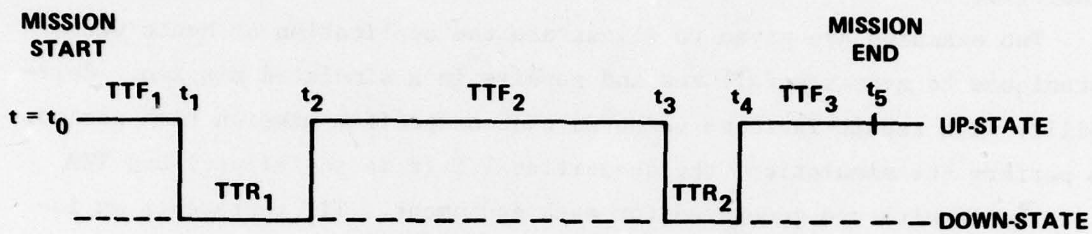
RN is a random number between 0 and 1.

To simulate a mission for a given scenario, a series of TTF's and TTR's is generated for each equipment. The sequence in which the times are generated will determine whether or not the simulated mission is successful.

3.2.2 Illustrative Example I

To illustrate the use of SIM3 in predicting reliability, consider operation of a system containing one equipment; when the equipment fails, so will the system.

The generation of TTF's and TTR's can be represented by a time line on a one-dimensional graph.



During the mission, equipment can assume two conditions or states. When the system is operating, it is referred to as being in an up-state; when the system is not operating due to failure, it is referred to as being in a down-state.

The mission starts at $t = 0$ with the system in the up-state. The first event is the generation of TTF_1 (time to the first failure)

$$TTF_1 = -MTBF \ln RN$$

$$t_0 + TTF_1 = t_1$$

At t_1, the system drops into the down-state for an interval TTR_1 (time for first repair)

$$TTR_1 = -MTTR \ln RN$$

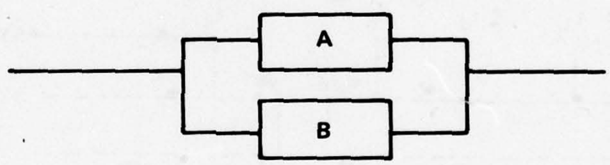
$$t_1 + TTR_1 = t_2$$

At t_2 the equipment is repaired and resumes operation. Similarly, TTF_2 and TTR_2 represent the second time to failure and time to repair. TTF_3, the third time to failure does not fail the system until after the mission ends at t_5 with the system in an up-state.

In this example, whenever the equipment failed (became inoperable), the system dropped into the down-state. In a system composed of two equipment in series, similar time-lines are generated. If either equipment fails, the system drops into the down-state, even though the other equipment is still functioning. If the two equipments are in parallel, the system might remain in operation if one equipment fails depending upon the mission requirements.

3.2.3 Illustrative Example II

This example further illustrates repair, up- and down-states, and other concepts used in this simulation. A system made up of two equipments A and B in parallel has the configuration

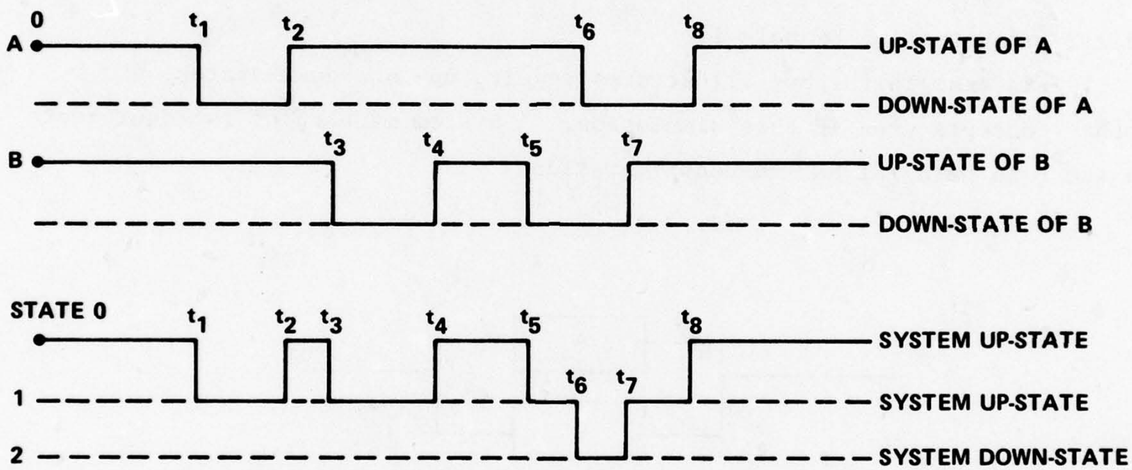


The operation of this configuration will be described over a mission extending from $t = 0$ to $t = t_8$. The operation of the system is specified by the up-state rule which states that, for a mission to be successful, either A or B must be in operation. If both A and B fail, the mission is aborted. The system can assume the following states:

State	Description
0	A and B both operational
1	Either A or B operational
2	A and B both failed

States 0 and 1 are considered up-states, since the system will operate successfully under either of these conditions. State 2 is a down-state.

This system is illustrated using two time-lines, one for A and one for B, and a third time-line indicating the state of the system as the result of failures and repairs of A and B.



Generating TTF and TTR as before, equipment A is down from t_1 to t_2, and B is down from t_3 to t_4. Since these intervals do not overlap, the system is in State 1 during both failures. At t_5, B goes down and the system again goes from State 0 to State 1. However, before B is repaired, A goes down (at t_6) and the system fails (State 2). At t_7, B is repaired and the system becomes operative again, proceeding to State 1. At t_8 A is repaired and the system is restored to State 0.

In this example, at least one of the two equipments is required for operation. When the system entered State 2 at t_6, the system failed and the mission aborted. However, if the two equipments had not failed at the same time, State 2 would not have been reached and the mission would have been successfully completed.

3.2.4 Mission Success

Since TTF's and TTR's are generated using random numbers, each simulated mission (replication) will be different. However, the actual states entered and the times at which they are entered are unimportant. What is important for either of the illustrative examples is the continuous operation of the entire system. If the system never enters a down-state, the mission is successful.

To obtain the reliability of a system, the simulation is executed with a given mission scenario and all the system failures are tabulated. At the end of a specified* number of replications, the reliability is computed.

$$\text{Reliability} = \frac{\text{Number of successful missions}}{\text{Total number of missions run}}$$

Reliability, the probability that the system will successfully perform the mission as specified in the scenario, is expressed as a statistical average, the percentage of successful missions.

3.3 SIM3 INPUT DESCRIPTION AND SPECIFIC EXAMPLE

Section 3.1 described the three types of input required by SIM3: mission scenario, system definition, and reliability data. This section describes the data in greater detail.

*The number of replications required to give a specific accuracy can be determined stastically or experimentally by performing sensitivity studies on the number of replications.

3.3.1 Mission Information

3.3.1.1 Scenario. The mission scenario is described in terms of the length of each phase, the system definition during that phase, and related information.

3.3.1.2 Abort Criteria. One of the characteristics of SIM3 which adds to its usefulness is the inclusion of abort criteria normally not available in analytic methods. These criteria are given in the form of three values (T_1, T_2, and T_3) specified for all subsystems and for the system itself, where $T_1 \leq T_2 \leq T_3$:

- T_1 is defined as negligible subsystem or system downtime. If the subsystem (or the system) is down for a time less than T_1, an equipment failure is not recorded and the mission does not abort.
- T_2 is defined as allowable sustained downtime. Only if a subsystem (or the system) is down for a time t and $t > T_2$ does the mission abort.
- T_3 is defined as allowable cumulative downtime. A subsystem (or the system) may be permitted several down periods (where t_i is a down period) during each phase, so long as the cumulative total of such downtimes does not exceed a specified limit. Thus if $T_1 < t_i < T_2$, the mission will not abort unless $\sum_{i=1}^{n} t_i > T_3$ where n is the number of down periods.

These three abort criteria enable a realistic simulation of operating conditions, since failure of one subsystem does not always cause mission abort. If these abort procedures are not desired in the simulation, or if comparison with analytic methods is preferred, the setting $T_1 = T_2 = T_3 = 0$ is used.

3.3.2 System Definition

Once the equipment in each phase of the mission has been specified, each equipment will be identified by a code which is entered into the program as input. Figure 1 illustrates the use of the code in a sample block diagram of a simple system. (See Appendix C for derivation of a

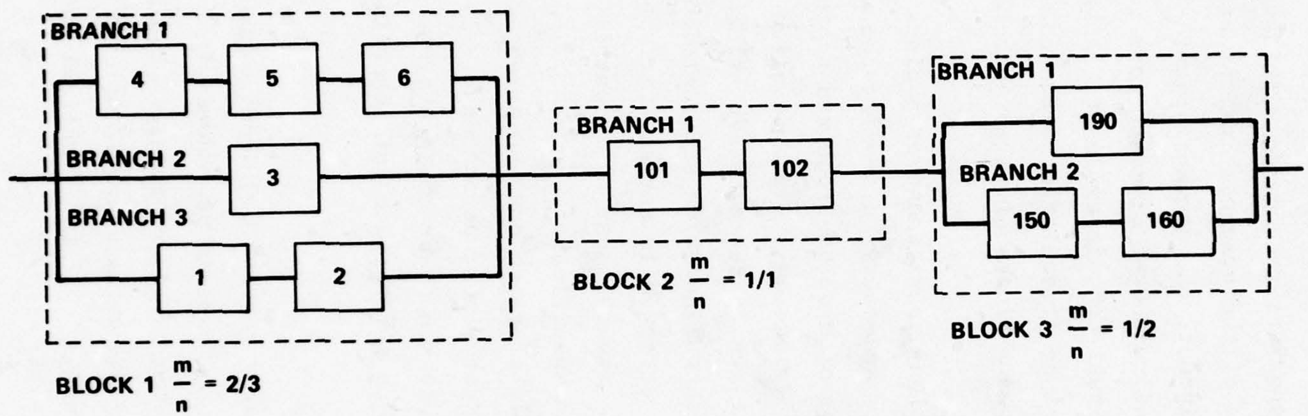


Figure 1 - Reliability Block Diagram for a Sample System

block diagram.) Each numbered rectangle in the diagram represents a separate equipment. The equipment numbers, a different number for each piece, are for identification purposes only and have no other significance.

As indicated in Figure 1 each alternative path in each block is a separate branch. The branches are renumbered within each block. Each equipment is located in one of the branches and the branches are arranged into logical blocks of equipment. As shown in Figure 1, series components form one block (Block 2) and redundant elements form two separate blocks (Blocks 1 and 3). A block must contain at least one branch and a branch must contain at least one piece of equipment. Branches are numbered consecutively from top to bottom. Blocks are also numbered consecutively from either direction.

A redundant configuration occurs whenever a block contains more than one branch. It then becomes necessary to specify the up-state rule for that block, i.e., the number of branches required for successful operation. If a block contains n branches and only m are required for system operation, the up-state rule is given in terms of m/n as shown in Figure 1.

The rules for mapping a configuration are as follows:

1. All blocks must be arranged in series.
2. All branches within a block must be in parallel. For each block an m/n up-state rule must be specified.
3. All equipment within a branch must be arranged in series.
4. Blocks in each subsystem must be consecutively numbered.
5. Branches within a block must be consecutively numbered.
6. Equipment numbers are for purposes of identification only.

3.3.3 Equipment Type Number

SIM3 was written initially for the CDC 3300* computer where limited size necessitated incorporating certain procedures into the program to decrease the amount of required computer storage. Equipments which have exactly the same MTBF, MTTR, and utilization factor (UF) are grouped into a type and assigned a type number. If two identical equipments are used

*SIM3 has recently been adapted to the CDC 6700 series computers.

in the same phase of a mission, each must be referred to by a different equipment number. If these two equipments carry the same type number, the identical character of the equipment is retained. When an identical equipment is used in different phases, the same equipment number and type may be used. Thus identical equipments are always referred to by the same type number, although they may have different equipment numbers.

3.4 SAMPLE CONFIGURATION

A sample problem may serve to clarify the concepts of type and equipment number, mission, and system definition as related to SIM3. A three-phase mission may be diagrammed along a horizontal line (Figure 2).

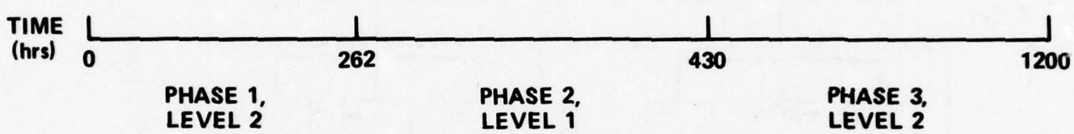


Figure 2 - Mission Profile for System A

In this example, the mission begins at t = 0 and ends at t = 1200. The mission specifies that the system will assume two levels of performance. The configuration for each level is shown in Figure 3. During Phases 1 and 3, Level 2 is required for operation; during Phase 2, Level 1 is required.

In Figure 3, the configuration is divided into four blocks; within each block each rectangle represents an equipment. Only the configuration in Block 1 changes between Levels 1 and 2. The configuration in Block 1 of Level 2 displays redundancy; that in Block 1 of Level 1 is arranged in series.

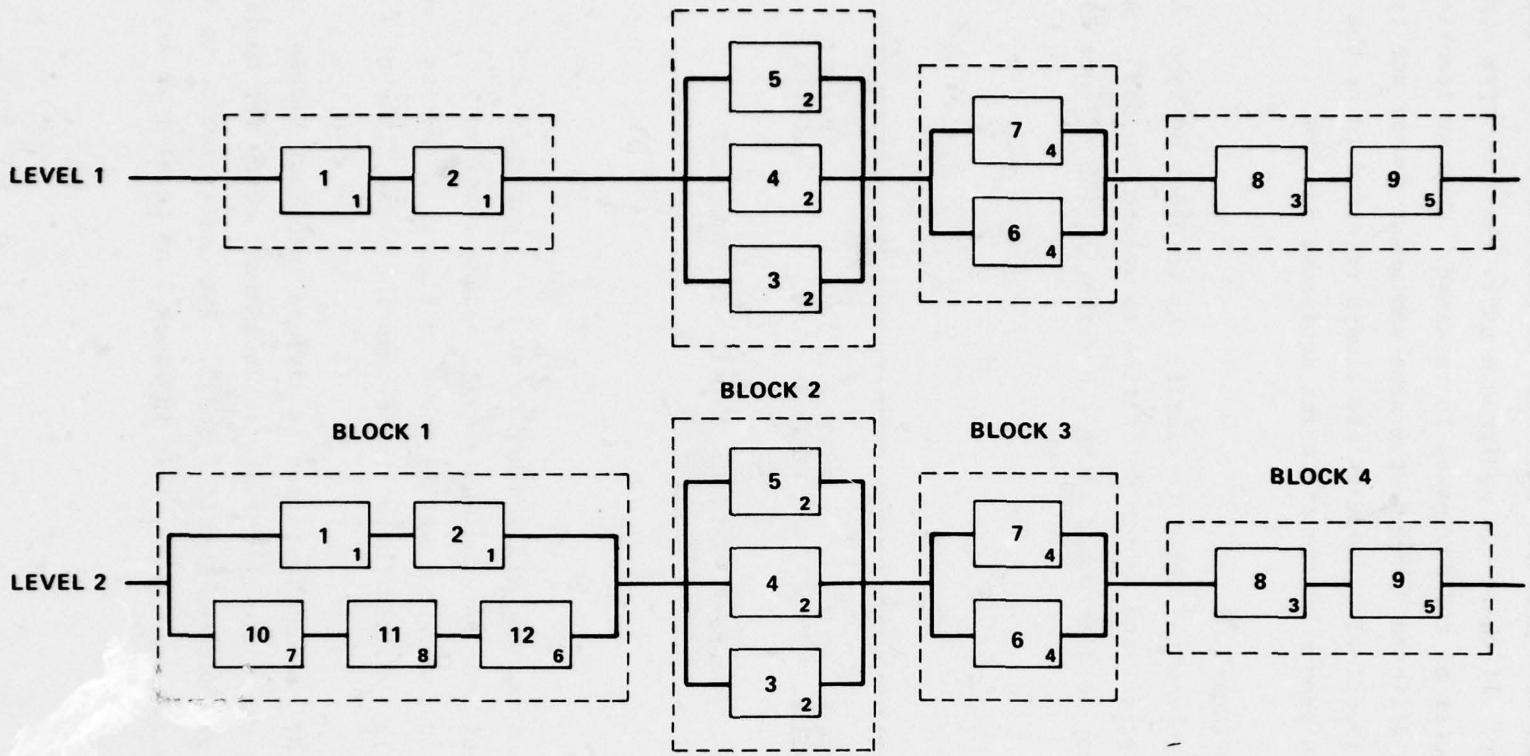


Figure 3 - Block Diagram of System A for Use with SIM3

The system diagrammed in Figure 3 contains eight types of equipment. The type number for each equipment is indicated in the lower right-hand corner of each rectangle. Block 1, Level 2 contains four different types of equipment and Level 1 contains two identical equipments, 1 type. In both Levels 1 and 2, Block 2 contains three type-2's in parallel and Block 3 contains two type-4's in parallel, both redundant configurations. Block 4 contains two different equipments in series.

When the equipment in a block is redundant, an up-state rule must be specified. For Block 2, assume m/n = 2/3 is specified i.e., two of the three branches in parallel must be operable for the subsystem to be operable. For Block 1, Level 2 and for Block 3, Levels 1 and 2, m/n = 1/2 is specified.

Appendix B describes the input deck setup for SIM3 and includes a sample printout.

SECTION 4

ANALYTIC RELIABILITY PREDICTION THEORY

This section discusses the analytical approach to reliability including the theory of the Poisson failure process and its incorporation into the analytic prediction method. Both exact and approximate solutions will be derived for solving the set of differential equations from which reliability and availability are computed.

4.1 THE STOCHASTIC MATRIX

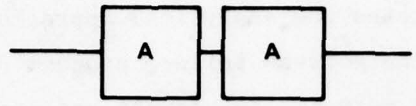
4.1.1 Theoretical Aspects

To compute the reliability of a system analytically, all the states that the system can assume must be identified. Up-states are defined as those states in which a system is operative. Down-states are those that occur when a system fails. The configuration of the equipment in the system determines up-states and down-states. For example, in a system composed of two identical equipments, called A, the following three states occur (A represents an equipment that is up and $\bar{A}$ an equipment that is down):

State	Symbol and Description
0	AA - both operational
1	$A\bar{A}$ - one operating and one failed
2	$\overline{AA}$ - both failed

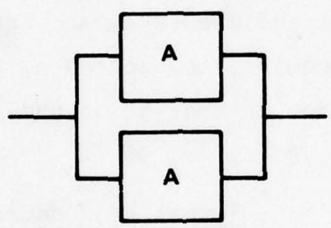
The probabilities for the three states are defined as P_0, P_1, and P_2. The system will always be in one of these three states; therefore, the sum of the three probabilities is 1. The probabilities can be determined knowing only what equipment exists in the system configuration, without knowledge of the arrangement itself. To determine the reliability from the probabilities, the configuration must be known.

Since the reliability is the probability that a system is in operational condition, only the up-states are used for computing the reliability. For example, if a system has the following configuration, consisting



as State 0 in which both A's are in operation. If either equipment fails, the system fails. The reliability of this configuration is P_0, the probability that both equipments are operating.

If the system configuration consists of two equipments arranged in parallel,



where redundancy is assumed, then the operation of only one equipment is required for system operation, so the up-states are 0 and 1. The reliability would equal $P_0 + P_1$, the probability that either or both of the equipments are operating.

For either system configuration, State 2 describes a condition of system failure. Since reliability is the probability that the system is up, P_2 is not considered in the reliability of either system configuration.

Once we have defined all the states in the system, a matrix, called a stochastic matrix, can be generated whose elements represent the transitions between these states. If N represents the number of states in the system,

the dimensions of the matrix are N×N. In the above example of two identical equipments, the stochastic matrix is dimensioned 3×3, containing nine elements.

In general, an N×N matrix has N^2 elements or transitions, but not all of them are allowed, for the following rules must be observed when using the Poisson failure process[7]:

• The probability of a transition in the interval t, t + Δt is $\lambda \Delta t$, where λ is the failure rate.

• The probability of more than one failure in the above interval is zero.

• The transition probabilities are independent of the state of the system.

Transitions can occur only between two adjacent states, e.g., from State 0 to State 1 or from 1 to 2, but not from 0 to 2. The probability of a transition from State 0 to State 2 is defined to be 0.

These rules and the method for deriving the transition probabilities are given by Sandler.[7] In the next sections the derivation of the stochastic matrix and the calculations of reliability and avilability from the stochastic matrix will be illustrated. Since these calculations are fairly straightforward, emphasis will be placed on the derivation of the stochastic matrix for various configurations.

4.1.2 Derivation of the Stochastic Matrix

The N×N stochastic matrix represents a set of N simultaneous differential equations (one equation for each state). In simple cases such as the one above, these equations can be solved exactly, but when the number of states becomes large, infinite series are needed for a solution. The accuracy desired in the series approximation can be specified. If the desired accuracy is obtained, the solution of the differential equations using series can then be considered exact.

For our first example of the construction of a stochastic matrix we shall examine the case of reliability without repair. Simple expressions

have been derived which are used to determine the reliability of a non-maintained system and a complicated derivation is unnecessary. However, for illustrative purposes, we shall indicate how these expressions can be determined by use of stochastic matrices.

4.1.3 Reliability without Repair

Each element in a stochastic matrix represents a transition from an initial state, represented vertically, to a final state, represented horizontally.

The stochastic matrix for a system consisting of two identical pieces of equipment for which repair is not available and for which the failure rate $\lambda = \frac{1}{MTBF}$ is as follows:

$$\begin{array}{c c} \underline{\text{Initial States}} & \underline{\text{Final States}} \\ & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 1-2\lambda & 2\lambda & 0 \\ 0 & 1-\lambda & \lambda \\ 0 & 0 & 1 \end{pmatrix} \end{array} \qquad (1)$$

The States 0, 1, and 2 have been defined for a system consisting of two pieces of equipment. The matrix of Equation (1) has nine elements. When a transition violates the rules represented in Section 4.1.1, a zero is entered. The 00 element represents a transition from an initial State 0 to a final State 0, i.e., the probability that both equipments will remain in operable condition through the interval dt. This transition represents the probability $(1-2\lambda)$ that neither equipment will fail. Element 01 represents the probability of transition from State 0 to 1, i.e., the probability that one of the equipments will fail. Since both equipments have the same probability of failure, the transition is represented by $\lambda + \lambda = 2\lambda$. The 02 element is zero since it is not a transition between two adjacent states. Element 10 is zero. Since there is no repair, the system cannot go from State 1 back to State 0. Element 11 represents the

probability that one of the pieces of equipment does not fail when the other already has failed and is $1 - \lambda$. Element 12 represents the probability that if one equipment has failed the second one will fail also, and is λ. Element 20 is zero for it represents a two-state transition. Element 21 is zero, since repair is not allowed. Element 22 is 1, for when both equipments have failed they will always remain failed (no repair).

4.1.4 Reliability with Repair

If repairs can be made, down-state items can be made operable, thus increasing the system reliability. Once we assume a maintained system, repairmen must be introduced. If one repairman is assigned to each equipment, an equipment that fails can be repaired immediately by a dedicated repairman. If two or more equipments are down at the same time, both can be repaired simultaneously by two dedicated repairmen. On the other hand, if the number of repairmen is less than the number of equipments in the system, the repair capability is reduced, decreasing the reliability.

The following matrix represents a system composed of two identical equipments in parallel with two repairmen, where μ is the repair rate.

$$\begin{array}{cc} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 1-2\lambda & 2\lambda & 0 \\ \mu & 1-(\lambda+\mu) & \lambda \\ 0 & 2\mu & 1-2\mu \end{pmatrix} \end{array} \qquad (2)$$

The repair rate is the reciprocal of the mean time to repair.

Some elements of this matrix are identical to those of the matrix in which repair was not considered. We shall examine only those elements that change with the introduction of repair. Element 10 is μ, the probability that one equipment is repaired. Element 11 is $1 - (\lambda+\mu)$, the probability that one equipment is not repaired and the other does not fail. Element 21 is μ, the probability that either equipment is repaired. And Element 22 is $1 - 2\mu$, the probability that neither equipment is repaired.

However, if only one repairman is available for the two equipments, Element 21 becomes μ and Element 22, $1 - \mu$. In this situation, only one equipment would be repaired at a time. The coefficient of μ represents the number of repairmen specified.

4.1.5 Standby Redundancy

If a system has a configuration with a + b identical equipments in parallel, and at least a equipments are required for continuous operation, the configuration is termed redundant. If the b redundant equipments are fully operative, active redundancy is present. If the b equipments are not operational until some of the a primary equipments fail, standby redundancy exists.

The stochastic matrix for a two-equipment standby redundant configuration (only one active equipment) with one repairman is

$$\begin{array}{cc} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 1-\lambda & \lambda & 0 \\ \mu & 1-(\lambda+\mu) & \lambda \\ 0 & \mu & 1-\mu \end{pmatrix} \end{array} \tag{3}$$

Element 00 becomes $1 - \lambda$, indicating the probability that one equipment will not fail. This is so, since only one equipment is active and capable of failing; the equipment in standby is inactive and therefore is not considered. The same argument holds for Element 01 which becomes λ.

4.2 SOLUTION METHODS FOR STATE PROBABILITIES AND RELIABILITIES

4.2.1 Exact Solution

To illustrate the exact analytic solution using the stochastic matrix, reliability without repair will be used. More complicated cases involving repair can also be solved analytically, but there are limitations. As the number of states becomes large, approximate methods must be used. This application will serve mainly to introduce the theory of the state probabilities.

Equation (1), the stochastic matrix for two identical equipments without repair, is repeated here:

$$\begin{pmatrix} 1-2\lambda & 2\lambda & 0 \\ 0 & 1-\lambda & \lambda \\ 0 & 0 & 1 \end{pmatrix} \tag{1}$$

Two operations must be performed to convert this matrix into usable form. First, 1's are subtracted from all the diagonal elements and then the transpose of the matrix is taken, resulting in

$$\begin{pmatrix} -2\lambda & 0 & 0 \\ 2\lambda & -\lambda & 0 \\ 0 & \lambda & 0 \end{pmatrix} \tag{4}$$

If $P_0(t)$, $P_1(t)$, and $P_2(t)$ are the probabilities of the system being in States 0, 1, and 2, then their relation with the stochastic matrix is

$$\begin{pmatrix} P_0'(t) \\ P_1'(t) \\ P_2'(t) \end{pmatrix} = \begin{pmatrix} P_0(t) \\ P_1(t) \\ P_2(t) \end{pmatrix} \begin{pmatrix} -2\lambda & 0 & 0 \\ 2 & -\lambda & 0 \\ 0 & \lambda & 0 \end{pmatrix} \tag{5}$$

where (') represents derivatives with respect to time.

Equation (5) breaks down into the following three simultaneous differential equations:

$$P_0'(t) = -2\lambda\, P_0(t) \tag{6}$$

$$P_1'(t) = 2\lambda\, P_0(t) - \lambda P_1(t) \tag{7}$$

$$P_2'(t) = \lambda P_1(t) \tag{8}$$

Since the system is fully operational at $t = 0$, the initial conditions are:

$$
\begin{aligned}
P_0(0) &= 1 \\
P_1(0) &= 0 \\
P_2(0) &= 0
\end{aligned}
\qquad (9)
$$

Using Laplace transforms to solve Equations (6), (7), and (8) gives the following expressions for the state probabilities:

$$P_0 = e^{-2\lambda t} \qquad (10)$$

$$P_1 = 2e^{-\lambda t} - 2e^{-2\lambda t} \qquad (11)$$

P_2 represents a down-state and is not used in the reliability computation.

If the two equipments are specified in series, the reliability results represent the probability that both equipments are operational, or

$$\text{Reliability} = P_0 = e^{-2\lambda t} \qquad (12)$$

Equation (12) could have been obtained by multiplying the reliabilities of the individual equipments together (i.e., $e^{-\lambda t} \times e^{-\lambda t}$). However, if the equipments are in parallel, then the reliability is the probability that at least one equipment is operational, stated as

$$\text{Reliability} = P_0 + P_1 = 2e^{-\lambda t} - e^{-2\lambda t} \qquad (13)$$

This expression could have been derived from the standard expression for the reliability of parallel equipments which for two equipments is

$$\text{Reliability} = 1 - (1-R)^2 \tag{14}$$

where R is the reliability of one equipment.

If we assume an exponential distribution $R = e^{-\lambda t}$, then

$$\text{Reliability} = 1 - (1-e^{-\lambda t})^2 \tag{15}$$

$$= 2e^{-\lambda t} - e^{-2\lambda t} \tag{16}$$

and Equation (16) is the same as Equation (13).

For a simple case like this in which the expressions are already known, Laplace transforms are not necessary. Their use is indicated for the more complicated analytical cases.

4.2.2 Approximate Solution

When the number of states becomes large, approximate solutions are required. Infinite series are introduced to facilitate a solution.

Let [A] represent the stochastic matrix of n dimensions and

$$[\dot{P}(t)] = (\dot{P}_0(t), \dot{P}_1(t), \dot{P}_2(t) \; . \; . \; . \; . \; \dot{P}_n(t)) \tag{17}$$

$$[P(t)] = (P_0(t), P_1(t), P_2(t) \; . \; . \; . \; . \; P_n(t)) \tag{18}$$

Then the matrix Equation (5) can be written as

$$[\dot{P}(t)] = [P(t)][A] \tag{19}$$

Also, let the initial probability state vector be specified as

$$[P(0)] = (P_0(0), P_1(0), \ldots P_n(0)) \tag{20}$$

so that the solution of the matrix equation is of the form

$$[P(t)] = e^{[A]t}\,[P_0] \tag{21}$$

To evaluate this expression, the infinite series expansion for the exponential function $e^{[A]t}$ is used.

$$e^{[A]t} = [I] + [A]t + \frac{[A]^2t^2}{2!} + \ldots \frac{[A]^jt^j}{j!}$$

$$= \sum_{j=0}^{\infty} \frac{[A]^jt^j}{j!} \tag{22}$$

In this series, t is the time variable and [I] is the n×n identity matrix (1's in the diagonal elements and zero's elsewhere).

The infinite series $e^{[A]t}$ can be evaluated for a specified number of terms providing t does not become much greater than $||A||$ (the magnitude of [A]). If $t >> ||A||$, the series will not converge. To make the solution of Equation (21) generally applicable, it is necessary to eliminate any possibility of nonconvergence. As an illustration, let us represent the matrix equation $[\dot{P}(t)] = [P(t)][A]$ in a different form as

$$\dot{y}(t) = Ay(t) \tag{23}$$

where y is an n-dimensional column vector and A is an n×n matrix.

To overcome the problem of nonconvergence when $t >> ||A||$, an iterative procedure is introduced which allows the evaluation of the series with an increment Δt of t, instead of t itself. The value of Δt is of the same order of magnitude as A in order to force the series to converge. The solution of Equation (23) is in the form

$$y(t) = e^{At} y_0 \tag{24}$$

This equation should hold (theoretically) for any value of t. Thus, if $t \rightarrow t + \Delta t$, the equation becomes

$$\begin{aligned} y(t+\Delta t) &= e^{A(t+\Delta t)} y_0 \\ &= e^{At} e^{A\Delta t} y_0 \\ &= e^{A\Delta t} e^{At} y_0 \end{aligned} \tag{25}$$

$$y(t+\Delta t) = e^{A\Delta t} y(t)$$

since $y(t) = e^{At} y_0$.

To determine $y(t+\Delta t)$, we need evaluate only the series

$$e^{[A]\Delta t} = [I] + [A]\Delta t + \frac{[A]^2 \Delta t^2}{2!} + \ldots \tag{26}$$

instead of e^{At}.

Consequently, to evaluate $y(t)$ for any t, the series $e^{[A]\Delta t}$ has only to be evaluated once.

The iteration process will be illustrated by indicating how the value of $y(t)$ for $t = T$ is obtained. T is a specified time interval (measured from $t = 0$) and is much larger than [A]. An increment Δt is chosen (a

value of 0.05 is presently used) and for simplicity we assume it will divide evenly into T so that on dividing $\frac{T}{\Delta t} = n$. The value of n represents the number of iterations required to obtain y(t). (Non-integer values of n can be handled in the actual calculations.)

Equation (25) is

$$y(t+\Delta t) = e^{A\Delta t}y(t)$$

Letting t = 0 and $y(0) = y_0$, the specified initial value of y(t)

$$y(\Delta t) = e^{A\Delta t}y_0 \tag{27}$$

Thus, with $e^{A\Delta t}$ and y_0 known, $y(\Delta t)$ may be obtained. Then, letting $y_0 = y(\Delta t)$, a value for $y(2\Delta t)$ can be obtained, and so on up to $y(n\Delta t)$.

The iteration process may be represented as

$$y(\Delta t) = e^{A\Delta t}y_0$$

$$y(2\Delta t) = e^{A\Delta t}y(\Delta t)$$

$$y(3\Delta t) = e^{A\Delta t}y(2\Delta t)$$

$$\vdots$$

$$y(n\Delta t) = e^{A\Delta t}y[(n-1)\Delta t]$$

and

$$y(T) = y(n\Delta t) \tag{28}$$

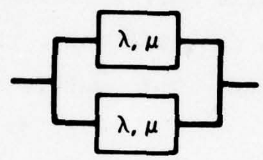
which gives the desired state probabilities represented by the vector y.

4.3 DETERMINATION OF RELIABILITY AND AVAILABILITY USING THE STOCHASTIC MATRIX

Once the stochastic matrix has been derived, availability as well as reliability can be obtained.

4.3.1 Stochastic Matrix Derivation

A configuration with two identical equipments in parallel and two repairmen may be diagrammed as



The stochastic matrix for such a system, shown as matrix (2), is converted into an operational form by subtracting 1 from each of the diagonal elements and transposing. The matrix becomes

$$\begin{pmatrix} -2\lambda & \mu & 0 \\ 2 & -(\lambda+\mu) & 2\mu \\ 0 & \lambda & -2\mu \end{pmatrix} \qquad (29)$$

The system can assume three states

State 0 - Both equipments operational

State 1 - One equipment operational, one equipment failed

State 2 - Both equipments failed

Only in States 0 and 1 will the system be operational. State 2 represents system failure.

4.3.2 Calculation of Availiability

In calculating the state probabilities when availability is desired, the entire stochastic matrix of dimensions 3×3 is used. The availability represents the sum of the probabilities of all up-states.

$$\text{Availability} = P_0 + P_1 \text{ (using entire stochastic matrix of dimension } n\times n) \tag{30}$$

4.3.3 Calculation of Reliability

In calculating the state probabilities when reliability is desired, an adjustment in the stochastic matrix is required. If NSF represents the number of up-states (in this case NSF = 2), then a new 2×2 matrix consisting of the up-states is formed. This matrix is

$$\begin{pmatrix} -2\lambda & \mu \\ 2\lambda & -(\lambda+\mu) \end{pmatrix} \tag{31}$$

derived by entering zeros in the last column of the original stochastic matrix. In the calculation of reliability, transitions from down-states to up-states are discarded. This rule comes from the definition of reliability, which states that the system must not be in a down-state during the specified time interval. Thus, the derivations of Elements 12 and 22 in matrices (2) and (3) were not required for the computation of reliability. They were described, since they are required in the availability calculations. Since only P_0 and P_1 are required, a 2×2 matrix is sufficient.

$$\text{Reliability} = P_0 + P_1 \text{ (derived from NSF}\times\text{NSF up-state matrix)} \tag{32}$$

4.3.4 Truncation Error

The use of an infinite series to solve the set of differential equations for the state probabilities may result in an approximate calculation. The question to be answered is, "How much of an approximation?"

As the number of terms used in the series to evaluate the quantity $e^{A\Delta t}$ increases, the results become more and more exact. In addition, the size of the increment Δt has an effect on the accuracy and must be considered.

Investigators at Naval Applied Science Laboratory (NASL) have developed equations to evaluate the truncation error, given the number of terms in the series and the increment. The truncation error represents the derivation of the series solution from the exact value. For instance, if we evaluate the series to the nth term we have

$$e^{A\Delta t} = \sum_{j=0}^{n} \frac{[A]^j \ [\Delta t]^t}{j!} \tag{33}$$

The truncation error T would be

$$T = \sum_{j=n+1}^{\infty} \frac{[A]^j \ [\Delta t]^t}{j!} \tag{34}$$

We want to determine T, given n and Δt. Equation (35) has been derived by NASL to provide an upper bound for T

$$T = \frac{\theta^{n+2}}{(n+2)!} \left[1+ \frac{\theta}{n+3} \sin\ (n\times\theta)\right] \tag{35}$$

where $\theta = Y_A \Delta t$

$Y_A = ||A||$, the magnitude of A

Δt is the time increment

T is the truncation error

n is the number of terms taken in the series

Since Δt is known and Y_A can be computed from the derived stochastic matrix, the truncation error can be found. Conversely, if T is specified, a combination of n and Δt can be found to satisfy that value.

At present $\Delta t = 0.5$ is used with a maximum error of 10^{-8} allowed. Starting with $n = 5$, n is increased until $T \leq 10^{-8}$. If T is already less than 10^{-8} when $n = 5$, five terms are used in the series.

SECTION 5

ANALYTIC RELIABILITY PREDICTION USING GEMJR

5.1 OVERVIEW

GEM is a comprehensive R/M computer model, developed at NASL and implemented on the CDC 6700 at DTNSRDC. GEM computes various elements of R/M from complex system definitions. Since GEM employs a user-type language, little knowledge of the computer or of programming is required, and the program is well-suited to users not desiring to get involved in the computer aspects of R/M calculations. However, because the user orientation of GEM necessitates a large computer program with its own compiler and function library, GEM cannot be run simultaneously with other programs. Consequently, a much smaller computer program that would compute elements of R/M was needed, and GEMJR was developed at DTNSRDC. This program, although not user oriented as GEM, can calculate many R/M quantities and has been used to investigate the feasibility of developing small but comprehensive versions of GEM for specific applications.

GEMJR incorporates the Poisson failure process, utilizing the theory developed in the previous section. It follows the lines of the original GEM program, incorporating similar theory and calculations. Because of its small size, GEMJR can be readily adapted to other computer programs. Although GEMJR was developed for a specific application, a user familiar with the basic theory can construct general programs (which will still be relatively small) to accommodate various system configurations and missions. A description and listing of GEMJR and its required input are given in Appendix D.

5.2 SAMPLE PROBLEM

GEMJR was constructed to calculate reliability and availability of the system configuration described in Section 3.4. The reliability block diagram and mission scenarios for GEMJR are identical to those given in Figure 3.

The block diagrams in the problem are straightforward; however, the specification of two levels of operation during the mission complicates

the reliability calculations. Each level of operation is associated with a different block diagram. Thus, when the level of operation changes, the block diagram changes. Each block diagram requires a different stochastic matrix. To compute reliability from a block diagram with changes in level of operation, the concept of interval reliability (i.e., reliability computed for a specific time interval) must be used to retain continuity of system states.

5.3 SYSTEM DEFINITION

5.3.1 Independence

If the concept of independence is assumed, the block diagram in Figure 3 can be partitioned to simplify the problem. The configuration can be divided into a series of smaller units called stages* which are simpler to handle than the entire configuration as a whole. The assumption of independence places certain restrictions on the problem. However, these restrictions are minor compared to benefits realized in the form of reduced effort to solve the problem.

To illustrate the concept of independence and the simplification obtained, the system states are described. When the Poisson process is used, the stochastic matrix is of dimension N×N, where N is the total number of states in the system. If M represents the number of <u>different</u> equipments in the block diagram, then

$$N \geq 2^M$$

When all equipments in the system are different, $N = 2^M$; otherwise the inequality holds, i.e., at least two equipments are similar. The total number of states decreases when more equipments are similar.

A look at Level 1 of the block diagram in Figure 3 shows that M = 9 so that

$$N = 2^9 = 512$$

---

*The stages are the same as the blocks defined in Figure 3.

Thus, a 512×512 stochastic matrix is needed to compute the reliability. If the configuration is divided into the four stages indicated, the largest matrix required is 32×32 (i.e., $32 = 2^5$), the size needed for Stage 1, Level 2, with 5 different pieces of equipment. Under the concept of independence, each stage is assumed independent of the others, i.e., all R/M quantities are computed separately for each stage, and each has its own repairman and standby equipment when applicable; there is no sharing between stages. The four stages are solved independently.

Complicated problems can be simplified by invoking independence. However, care must be taken to avoid distorting the problem by assuming an independence that does not apply.

5.3.2 System Configuration

All series equipments can be treated alike, whether or not repairs are made, for the system fails when any series equipment fails. Even if repairs are allowed, the system will be down during the repair period and the mission will be aborted. Consequently, the reliability of Stage 4, which consists only of series equipments, can be computed from the following standard formula for reliability

$$\text{Reliability} = e^{-\lambda t} \tag{36}$$

where λ is the failure rate and t is the failure time.

Equation (36) makes no assumptions or approximations. The reliability of these series equipments is independent of the number of repairmen assigned to them. The mission specifies up-state for Equipments 3, 4, 5, 6, and 7. Because of the group reference to these equipments, in the mission we refer to Equipments 3, 4, and 5 as Stage 2, and Equipments 6 and 7 as Stage 3. Since Stages 2 and 3 are independent of each other, repairmen cannot be shared. Only the equipment configuration of Stage 1 changed during the mission.

The assumption of independence permits each stage to be identified with a simpler stochastic matrix. Stage 1 has a 5×5 matrix, Stage 2 a 4×4 matrix, and Stage 3 a 3×3 matrix.

If there are M different equipments in a stage, then 2^M different states are possible. If some or all of the equipments are identical, fewer than 2^M states exist. As seen in Figure 3, both Stages 2 and 3 are composed of identical items within the stage, so the two stages have fewer than 2^M states.

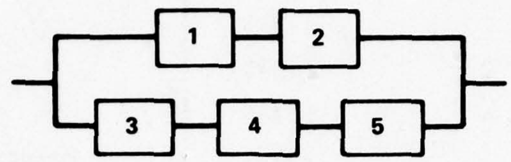
Stages 2 and 3 could have been treated as a single stage consisting of five pieces of equipment. This expanded stage would have involved a total of 12 different states, resulting in a more complicated derivation of the stochastic matrix. At the same time, however, such an expanded stage would have added flexibility by allowing the sharing of repairmen over these five equipments as a group.

The configurations of Stages 2 through 4 are the same in the different phases so the same expression for computing their reliabilities holds throughout the mission. The configuration of Stage 1 changes during the mission, however, as Phase 1 of Stage 1 is composed of five equipments, Phase 2 of two equipments, and Phase 3 of five equipments. This configuration change alters the stochastic matrix so, instead of using the standard reliability expression (see Section 5.5), the concept of interval reliability must be introduced to compute the reliability in Phases 2 and 3. Although Stage 1 contains only two equipments during Phase 2, it is assumed to contain (for calculation purposes only) five different equipments at all times, for a total of 32 possible states.

Because the stages are independent, their reliabilities, which are obtained separately, must be multiplied together to get the reliability of the total system during that phase. The reliability and availability for the entire mission are obtained by multiplying the reliability and availability of the different phases together.

5.4 STAGE 1 CONFIGURATION

Stage 1 can be represented by the following diagram



Each equipment is numbered and has associated with it failure rates, λ_1, λ_2 ... λ_5 and repair rates μ_1, μ_2 ... μ_5. The number of states in the stage is $2^5 = 32$, as listed in Table 1. The 32 states describe all the possible conditions of Stage 1 composed of equipments 1, 2, 3, 4, and 5. These equipment numbers are different from those in Figure 3 but are used for simplicity. In each state each equipment can be either up, i.e., operative, or down, i.e., failed. An equipment number without a bar indicates that the equipment is operational; an equipment number with a bar indicates that the equipment has failed. The configuration represents a type of redundancy in that the up-state, representing non-failure of the system, requires that either equipments 1 and 2 are operative or that equipments 3, 4, and 5 are operative, but not both sets at once. States 0 to 10 represent up-states and the rest down-states. The non-zero elements of the entire stochastic matrix are given in Table 2. To compute reliability with repair, only the up-states are considered. To compute interval reliability and availability, all the stages in the stochastic matrix must be considered.

5.5 R/M DEFINITIONS

In this section the terms reliability, availability, and interval reliability will be defined with respect to the stochastic matrix, state probabilities, and number of up-states.

TABLE 1 - POSSIBLE STATES FOR STAGE 1

State	Configuration	
0	12345	
1	$\overline{1}2345$	
2	$1\overline{2}345$	
3	$\overline{12}345$	
4	$12\overline{3}45$	
5	$123\overline{4}5$	Up-states
6	$1234\overline{5}$	
7	$12\overline{34}5$	
8	$123\overline{45}$	
9	$12\overline{3}4\overline{5}$	
10	$12\overline{345}$	
11	$\overline{1}2\overline{3}45$	
12	$\overline{1}23\overline{4}5$	
13	$\overline{1}234\overline{5}$	
14	$1\overline{23}45$	
15	$1\overline{2}3\overline{4}5$	
16	$1\overline{2}34\overline{5}$	
17	$\overline{123}45$	
18	$\overline{12}3\overline{4}5$	
19	$\overline{12}34\overline{5}$	
20	$\overline{1}2\overline{34}5$	
21	$1\overline{234}5$	Down-states
22	$\overline{1}23\overline{45}$	
23	$1\overline{2}3\overline{45}$	
24	$\overline{1}2\overline{3}4\overline{5}$	
25	$1\overline{23}4\overline{5}$	
26	$\overline{1}2\overline{345}$	
27	$1\overline{2345}$	
28	$\overline{12}3\overline{45}$	
29	$\overline{123}4\overline{5}$	
30	$\overline{1234}5$	
31	$\overline{12345}$	

TABLE 2 - NON-ZERO ELEMENTS FOR STAGE 1 STOCHASTIC MATRIX

TABLE 2A - DIAGONAL ELEMENTS

Transition	Element	Transition	Element
0,0	$1 - (\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5)$	16,16	$1 - (\lambda_1+\lambda_3+\lambda_4+\mu_2+\mu_5)$
1,1	$1 - (\lambda_2+\lambda_3+\lambda_4+\lambda_5+\mu_1)$	17,17	$1 - (\lambda_4+\lambda_5+\mu_1+\mu_2+\mu_3)$
2,2	$1 - (\lambda_1+\lambda_3+\lambda_4+\lambda_5+\mu_2)$	18,18	$1 - (\lambda_3+\lambda_5+\mu_1+\mu_2+\mu_4)$
3,3	$1 - (\lambda_3+\lambda_4+\lambda_5+\mu_1+\mu_2)$	19,19	$1 - (\lambda_3+\lambda_4+\mu_1+\mu_2+\mu_5)$
4,4	$1 - (\lambda_1+\lambda_2+\lambda_4+\lambda_5+\mu_3)$	20,20	$1 - (\lambda_2+\lambda_5+\mu_1+\mu_3+\mu_4)$
5,5	$1 - (\lambda_1+\lambda_2+\lambda_3+\lambda_5+\mu_4)$	21,21	$1 - (\lambda_1+\lambda_5+\mu_2+\mu_3+\mu_4)$
6,6	$1 - (\lambda_1+\lambda_2+\lambda_3+\lambda_4+\mu_5)$	22,22	$1 - (\lambda_2+\lambda_3+\mu_1+\mu_4+\mu_5)$
7,7	$1 - (\lambda_1+\lambda_2+\lambda_5+\mu_3+\mu_4)$	23,23	$1 - (\lambda_1+\lambda_3+\mu_2+\mu_4+\mu_5)$
8,8	$1 - (\lambda_1+\lambda_2+\lambda_3+\mu_4+\mu_5)$	24,24	$1 - (\lambda_2+\lambda_4+\mu_1+\mu_3+\mu_5)$
9,9	$1 - (\lambda_1+\lambda_2+\lambda_4+\mu_3+\mu_5)$	25,25	$1 - (\lambda_1+\lambda_4+\mu_2+\mu_3+\mu_5)$
10,10	$1 - (\lambda_1+\lambda_2+\mu_3+\mu_4+\mu_5)$	26,26	$1 - (\lambda_2+\mu_1+\mu_3+\mu_4+\mu_5)$
11,11	$1 - (\lambda_2+\lambda_4+\lambda_5+\mu_1+\mu_3)$	27,27	$1 - (\lambda_1+\mu_2+\mu_3+\mu_4+\mu_5)$
12,12	$1 - (\lambda_2+\lambda_3+\lambda_5+\mu_1+\mu_4)$	28,28	$1 - (\lambda_3+\mu_1+\mu_2+\mu_4+\mu_5)$
13,13	$1 - (\lambda_2+\lambda_3+\lambda_4+\mu_1+\mu_5)$	29,29	$1 - (\lambda_4+\mu_1+\mu_2+\mu_3+\mu_5)$
14,14	$1 - (\lambda_1+\lambda_4+\lambda_5+\mu_2+\mu_3)$	30,30	$1 - (\lambda_5+\mu_1+\mu_2+\mu_3+\mu_4)$
15,15	$1 - (\lambda_1+\lambda_3+\lambda_5+\mu_2+\mu_4)$	31,31	$1 - (\mu_1+\mu_2+\mu_3+\mu_4+\mu_5)$

Note: Under Transition first value is initial state, second value is final state.

TABLE 2 (Continued)

TABLE 2B - NON-DIAGONAL ELEMENTS

Transition	Element	Transition	Element	Transition	Element	Transition	Element	Transition	Element	Transition	Element	Transition	Element
0,1	λ_1	5,0	μ_4	10,7	μ_5	15,2	μ_4	20,7	μ_1	25,9	μ_2	30,17	μ_4
0,2	λ_2	5,7	λ_3	10,8	μ_3	15,5	μ_2	20,11	μ_4	25,14	μ_5	30,18	μ_3
0,4	λ_3	5,8	λ_5	10,9	μ_4	15,18	λ_1	20,12	μ_3	25,16	μ_3	30,20	μ_2
0,5	λ_4	5,12	λ_1	10,26	λ_1	15,21	λ_3	20,26	λ_5	25,27	λ_4	30,21	μ_1
0,6	λ_5	5,15	λ_2	10,27	λ_2	15,23	λ_5	20,30	λ_2	25,29	λ_1	30,31	λ_5
1,1	μ_1	6,0	μ_5	11,1	μ_3	16,2	μ_5	21,7	μ_2	26,10	μ_1	31,26	μ_2
1,0	λ_2	6,8	λ_4	11,4	μ_1	16,6	μ_2	21,14	μ_4	26,20	μ_5	31,27	μ_1
1,11	λ_3	6,9	λ_3	11,17	λ_2	16,19	λ_1	21,15	μ_3	26,22	μ_3	31,28	μ_3
1,12	λ_4	6,13	λ_1	11,20	λ_4	16,23	λ_4	21,27	λ_5	26,24	μ_4	31,29	μ_4
1,13	λ_5	6,16	λ_2	11,24	λ_5	16,25	λ_3	21,30	λ_1	26,31	λ_2	31,30	μ_5
2,0	μ_2	7,4	μ_4	12,1	μ_4	17,3	μ_3	22,8	μ_1	27,10	μ_2		
2,3	λ_1	7,5	μ_3	12,5	μ_1	17,11	μ_2	22,12	μ_5	27,21	μ_5		
2,14	λ_3	7,10	λ_5	12,18	λ_2	17,14	μ_1	22,13	μ_4	27,22	μ_3		
2,15	λ_4	7,20	λ_1	12,20	λ_3	17,29	λ_5	22,26	λ_3	27,25	μ_4		
2,16	λ_5	7,21	λ_2	12,22	λ_5	17,30	λ_4	22,28	λ_2	27,31	λ_1		
3,1	μ_2	8,5	μ_5	13,1	μ_5	18,1	μ_4	23,8	μ_2	28,18	μ_5		
3,2	μ_1	8,6	μ_4	13,6	μ_1	18,12	μ_2	23,15	μ_5	28,19	μ_4		
3,17	λ_3	8,10	λ_3	13,19	λ_2	18,15	μ_1	23,16	μ_4	28,22	μ_2		
3,18	λ_4	8,22	λ_1	13,22	λ_4	18,28	λ_5	23,27	λ_3	28,23	μ_1		
3,19	λ_5	8,23	λ_2	13,24	λ_3	18,29	λ_3	23,28	λ_1	28,31	λ_3		
4,0	μ_3	9,4	μ_5	14,2	μ_3	19,3	μ_5	24,9	μ_1	29,17	μ_5		
4,7	λ_4	9,5	μ_3	14,4	μ_2	19,13	μ_2	24,11	μ_5	29,19	μ_3		
4,9	λ_5	9,10	λ_4	14,17	λ_1	19,16	μ_1	24,13	μ_3	29,24	μ_2		
4,11	λ_1	9,24	λ_1	14,21	λ_4	19,28	λ_4	24,27	λ_4	29,25	μ_1		
4,14	λ_2	9,25	λ_2	14,25	λ_5	19,29	λ_3	24,29	λ_2	29,31	λ_4		

5.5.1 Reliability

A system is assumed to be in an up-state at the start of the mission, t = 0. The reliability at any time t (greater than t = 0) is defined as the probability that the system will not fail up to that time, t. During the time period t = 0 to t, redundant equipments can be repaired. Since reliability is the probability that the system will remain in an up-state (will not fail), transitions from up-states to down-states (system failure) are not allowed in the calculations. Therefore, only the abbreviated up-state version of the stochastic matrix is considered. From the up-state matrix, the state probabilities (for only the up-states) over the first phase can be computed. The initial state probabilities at t = 0 are specified as P(1)* = 1, and the rest, P(2) to P(11), zero, signifying that the system is fully up at t = 0. The state probabilities over Phase 2 are computed with respect to the new initial state probabilities, which are the state probabilities computed over Phase 1. Finally, state probabilities over Phase 3 are computed with respect to those from Phase 2. The up-state probabilities computed over Phase 3 (remember, they are a function of the state probabilities at Phases 1 and 2) are added to give the reliability for the mission, the probability that the system will be in an up-state condition.

5.5.2 Availability

If the system is assumed to be up at t = 0, the availability is defined as the probability that the system will be up at any time t greater than t = 0. The system could have failed and been restored many times in the interval (0,t), but the important consideration is the state of the system at time t.

Since the availability is a function of the time that the system is in both states (for the system can enter an up-state from a down-state and conversely), the entire stochastic matrix must be used in calculations. Therefore, all the state probabilities must be calculated to obtain the

---

*P(1) represents the probability that the system will be in State 0, similarly P(2) represents the probability that the system will be in State 1 and so on.

availability. The state probabilities over each phase are calculated as a function of the initial probabilities for that phase. The initial probability for other than the first phase is the final probability of the preceding phase. All the state probabilities at the end of Phase 3 are added to give the availability.

5.5.3 Interval Reliability

It is assumed that the system is completely operational at $t = 0$ and that, in the interval from $t = 0$ to t_1, system failures and repairs can take place. But, during the interval t_1 to t_2, only redundant items can be repaired if they fail. The interval reliability $R(t_1,t_2)$ of a system is defined as the joint probability that the system will be up at time t_1 and remain up until time t_2 which is greater than t_1. The concepts of both availability from 0 to t_1 and reliability from t_1 to t_2 are involved.

To calculate the interval reliability $R(t_1,t_2)$, the entire stochastic matrix must be used to obtain all the state probabilities over the interval $[0,t_1]$. Then only the up-state probabilities are used to compute the interval reliability over $[t_1,t_2]$.

5.6 CALCULATION OF RELIABILITY AND AVAILABILITY OF STAGE 1

5.6.1 Phase 1

To calculate reliability and availability for Stage 1, the following quantities are used:

Variables	Definitions
A	Stochastic matrix dimensioned 32×32
A'	Up-state stochastic matrix of dimension 11×11
P(0)	Column vector of state probabilities at $t = 0$ (Initial conditions)
P(t)	$(P_0(t), P_1(t) \ldots P_{31}(t))$, Column vector of all state probabilities at time t (32 elements)
P'(t)	Column vector of up-state probabilities (11 elements)

Δt Time increment

t_1 Time at end of Phase 1

n_1 $\frac{t_1}{\Delta t}$ = number of iterations required for first phase

To obtain the state probabilities at the end of Phase 1, n_1 iterations are necessary. The stochastic matrix for Stage 1 is given in Table 2. As shown in Equation (26), the following operations are required

$$
\begin{aligned}
P(\Delta t) &= e^{A\Delta t}P(0) \\
P(2\Delta t) &= e^{A\Delta t}P(\Delta t) \\
&\vdots \\
P(t_1) = P(n_1\Delta t) &= e^{A\Delta t}P((n-1)\Delta t)
\end{aligned}
\tag{37}
$$

Each line or iteration represents a matrix multiplication. The state probabilities are represented by a column vector and $e^{A\Delta t}$ is a square matrix so that multiplying a matrix by a column vector results in a column vector, i.e., the new state probabilities. After n_1 iterations, the state probabilities are known at t_1, and by combining them we get availability.

$$\text{Availability} = \sum_{i=0}^{31} P_i(t_1) \tag{38}$$

If the same procedure is followed and Matrix A is replaced with Matrix A' in Equation (37), we obtain reliability by summing the up-state probabilities.

Thus

$$P'(\Delta t) = e^{A'\Delta t}P(0)$$

$$P'(2\Delta t) = e^{A'\Delta t}P'(\Delta t)$$

$$\vdots$$

$$P'(t_1) = P(n_1\Delta t) = e^{A\Delta t}P'((n_1-1)\Delta t)$$

From P' and A' in the iteration Equation (37) and the up-state probabilities at t_1 (column vector), P' can be determined. The reliability at t_1 is determined by summing the elements of vector P'.

$$\text{Reliability} = \sum_{i=0}^{10} P_i'(t_1) \tag{39}$$

These calculations are used for the first phase of the mission. The configuration change of Phase 2 requires the calculation of interval reliability as described in the next section.

5.6.2 Phase 2

The configuration change from Phase 1 to Phase 2 makes it necessary to compute interval reliability; two changes must be made in the calculations as performed for Phase 1.

The configuration change during the phase change results in a change in the number of up-states, causing an alteration in the stochastic matrix. In Stage 1, the Phase 1 configuration consists of five equipments. The Phase 2 configuration consists of only two. Thus, there has been an actual decrease in the number of possible states from 2^5 to 2^2. However, use of the Poisson process requires that the number of possible states remain constant throughout the mission. Consequently, we must assume that there

are five equipments in Phase 2, even though only two equipments exist. Only the following eight states, which reflect the probability that both Equipments 1 and 2 are up, are considered in the reliability calculation.

$$12345$$
$$12\bar{3}45$$
$$123\bar{4}5$$
$$1234\bar{5}$$
$$12\overline{34}5$$
$$123\overline{45}$$
$$12\bar{3}4\bar{5}$$
$$12\overline{345}$$

By specifying these up-states and eliminating up-states $\bar{1}2345$, $1\bar{2}345$, and $\overline{12}345$ from Phase 2, we imply that the failures of only equipments 1 and 2 affect the reliability. Although the configuration has not changed physically, we have arranged the calculations so that the configuration actually consists of only two equipments in series.

The mathematical procedures used require that the down-states follow the up-states in the original stochastic matrix. Therefore the original stochastic matrix must be rearranged so that the first eight rows contain the up-states specified.

The second change, necessitated by the configuration change involves the calculation of interval reliability.

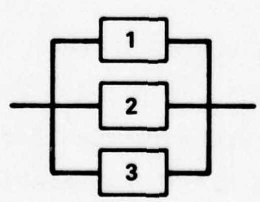
At the beginning of the mission, t = 0, initial values for the state probabilities are specified as $P_0(0) = 1$, $\sum_{i=1}^{31} P_i(0) = 0$. The 32 probabilities are computed at the end of Phase 1, using the total stochastic matrix, and the availability is computed. These probabilities become the initial values of the state probabilities for computing both availability and reliability during Phase 2. The availability is computed using the rearranged total stochastic matrix with all 32 initial values. The

reliability is computed using the abbreviated and rearranged stochastic matrix (8×8 matrix consisting only of up-states) with the first eight initial values being those for the state probabilities. When computing reliability with repair, the initial values of the reliability calculations are obtained from the previous reliability calculation (last phase). However, because we are calculating interval reliability, the initial values are obtained from the preceding availability calculations. Also, the resulting interval reliability is the reliability for the specified interval phase only. To obtain the reliability through Phase 2, the reliability of Phase 1 must be multiplied by the interval reliability for Phase 2. The same process is used again to compute the interval reliability over Phase 3. The original stochastic matrix is used because the configuration reverts to the original configuration of Phase 1.

5.7 CALCULATION OF RELIABILITY FOR STAGES 2, 3, AND 4

An analysis similar to that used for Stage 1 is used for Stages 2 and 3, the only difference being in the derivation of the stochastic matrix. For these stages, reliability with repair is calculated throughout the mission using the straightforward iteration process described in the last section.

The configuration of Stage 2, consisting of three identical equipments arranged in parallel, is



These are four possible states:

State	Description
0	All equipments operational
1	Two equipments operational
2	One equipment operational
3	All equipments fail

With one repairman for each equipment and active redundancy, the stochastic matrix is

$$\begin{pmatrix} 1-3\lambda & 3\lambda & 0 & 0 \\ \mu & 1-(2\lambda+\mu) & 2\mu & 0 \\ 0 & 2\mu & 1-(\lambda+2\mu) & \lambda \\ 0 & 0 & 3\mu & 1-3\mu \end{pmatrix} \tag{40}$$

The probabilities $P_1(t)$ can be determined up to any time t. Reliability and availability are obtained by summing the probabilities of the up-states at the end of each phase.

The up-state rules specify that two out of three pieces must be operational, i.e., only States 0 and 1 contribute to the reliability. Thus

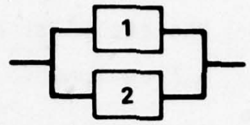
$$\text{Availability} = P_0(t) + P_1(t) \tag{41}$$

$$\text{Reliability} = P_0'(t) + P_1'(t) \tag{42}$$

The up-state 2×2 matrix used to compute reliability is

$$\begin{pmatrix} 1-3\lambda & 3\lambda \\ \mu & 1-(2\lambda+\mu) \end{pmatrix} \tag{43}$$

The configuration of Stage 3 consists of two identical equipments arranged in parallel



A repairman is assigned to each equipment. The states possible are

State	Description
0	All equipments operational
1	One equipment operational
2	All equipments fail

The stochastic matrix is

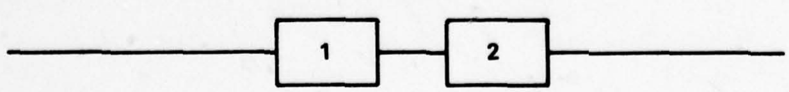
$$\begin{pmatrix} 1-2\lambda & 2\lambda & 0 \\ \mu & 1-(\lambda+\mu) & \lambda \\ 0 & 2\mu & 1-2\mu \end{pmatrix}$$

Since the up-state rules specify that one of two equipments must be up for operation, States 0 and 1 are up-states and the following notation applies

$$\text{Availability} = P_0(t) + P_1(t)$$

$$\text{Reliability} = P_0'(t) + P_1'(t)$$

The configuration of Stage 4 consists of two different equipments arranged in series



The simple expression for two identical equipments in series, Equation (12), can be applied to compute the reliability of Stage 4.

$$\text{Reliability} = e^{-\lambda t} \cdot e^{-\lambda t} = e^{-2\lambda t} \tag{12}$$

Since Stage 4 contains two different equipments, Equation (12) becomes

$$\text{Reliability} = e^{-\lambda_1 t} \cdot e^{-\lambda_2 t} \tag{45}$$

$$= e^{-(\lambda_1+\lambda_2)t} \tag{46}$$

5.8 CALCULATION OF RELIABILITY FOR TOTAL SYSTEM

The reliability and availability for each phase and each stage, and thus for the total mission, can be calculated. We shall outline the procedure for obtaining the reliability of the total system.

To obtain the reliability (R) for Stages 2 and 3, R_2 and R_3, respectively, we use their stochastic matrices, perform $\frac{t_2}{\Delta t}$ iterations, and sum the up-state probabilities at t_2.

Stage 1 is complicated by the configuration change in Phase 2. The state probabilities at the end of Phase 1, $t = t_1$, obtained through $\frac{t_1}{\Delta t}$ iterations, are called R_{11}. For Phase 2, we use the concept of interval reliability. The state probabilities at $t = t_2$ are obtained after $\frac{t_2-t_1}{\Delta t}$ iterations. Summing the up-states gives the interval reliability (IR) over t_1t_2, Phase 2, which is $IR(t_1t_2)$. The reliability for Stage 1 up through t_2 is

$$R_1 = R_{11}(t_1) \times IR(t_1t_2) \tag{47}$$

Finally the total reliability for Phase 2 is

$$R_{Phase\ 2} = R_1(t_2)\ R_2(t_2)\ R_3(t_2)\ R_4(t_2) \quad (48)$$

The same procedure is used to obtain the reliability at the end of Phase 3 (end of mission) at $t = t_3$, except that

$$R_3 = R_{11} \times IR(t_1 t_2) \times IR(t_2 t_3) \quad (49)$$

SECTION 6

COMPARISON OF R/M PREDICTION METHODS

Both the simulation and analytic reliability prediction methods have advantages and disadvantages so that the choice of the best program for a given situation is not obvious. The characteristics of the two programs are compared so that the user can make an informed decision.

6.1 SIMULATION METHOD

Before the reliability of a system can be calculated, a block diagram of the system must be constructed and the appropriate data obtained. SIM3 uses Monte Carlo techniques to simulate the operation of a system. The possible outcomes of such a simulated mission are mission success or mission abort. To calculate the reliability of the system, many missions must be simulated.

The reliability is computed stochastically as the number of successful missions is divided by the total number of missions. Since a prediction based on simulation is nondeterministic, the results of each computer run will be somewhat different. However, as more missions are simulated, the statistical prediction should approach that obtained from analytic (or deterministic) methods.

6.1.1 Advantages

The nature of the simulation process allows some highly complicated applications to be solved with relative ease.

First, SIM3 records the number of times each piece of equipment fails. These results are tabulated separately for each simulated mission and any equipment with a high failure record can be identified. Steps can be taken to improve the performance of the equipment and thus increase system reliability.

Second, SIM3 can handle system configuration changes more easily than can the analytic method since interval reliability is more easily calculated.

Finally, SIM3 provides for the specification of allowable sustained downtime. When a subsystem (or the entire system) fails and is down for a time less than the sustained downtime the abort criteria are not violated. This feature is not incorporated in analytic methods.

6.1.2 Disadvantages

The statistical nature of simulation means that final results are obtained by averaging. (It is possible to predict the actual number of mission simulations required to satisfy a given confidence limit.) As the system gets larger and more equipments are added, each simulated mission requires more computer time to run, and simulation of large systems can become quite expensive.

Another shortcoming of SIM3 is that it does not compute time-dependent availability, although steady-state availability can be computed. Also, simulation predictions are generally not as flexible as analytic methods in the consideration of the several aspects of maintainability; repairmen, standbys, redundancies, and spare parts cannot easily be considered.

Finally, the statistical results of simulation are not as exact as analytical results. The simulation prediction results approach the exact results as the number of simulated missions increases, but in some cases it is impractical to run enough missions to achieve desired accuracy. Thus the results usually deviate from the actual value by a few percent.

6.2 ANALYTIC METHOD

The theory of analytic prediction is covered in detail in Section 4. The Poisson failure process is used to develop stochastic matrices from which a set of simultaneous differential equations is generated. Solving these equations gives a one-dimensional array whose elements are the state probabilities. The more equipment to be considered, the greater the number of states and thus the greater the number of calculations involved.

6.2.1 Advantages

An analytic prediction is deterministic in that the results are always the same providing the input has not changed. There is no variation in the prediction as there is with simulation programs. Because of this exactness in results, an analytic process can be used when small changes in parameters are required. For instance, assume we want to observe the effect of adding additional redundant equipment in a configuration. Because our analytic prediction is exact, even small changes will usually be reflected in the answer. With the simulation prediction, such small changes might be indistinguishable from the normal variation in the results.

The primary input in an analytic prediction is the stochastic matrix. Once it is derived, the remaining calculations are routine. Many different aspects of maintenance can be incorporated by altering various elements of the stochastic matrix. Thus, repairmen and standby equipment can easily be considered. In addition, variations in the number of repairmen and standby equipment can be made, enabling tradeoffs with system maintenance cost and reliability.

6.2.2 Disadvantages

Because all the states that a system can assume must be identified in the analytic method, the size of the system for which reliability and availability are calculated is limited. The number of different states that a system can assume (as reflected by the dimensions of the stochastic matrix) is represented by 2^N, where N is the number of different pieces of equipment. If N = 5, the number of states possible is $2^5 = 32$, and the associated stochastic matrix would be dimensioned at 32 by 32. The derivation of this matrix is not a simple task. One can imagine the work required when N is even larger. Thus the subsystems must be kept to a "reasonable" size to keep the volume of calculations down. If N is unmanagably large, other methods which may include simulation programs, must be used.

The failure-prone items can be more easily identified using simulation methods. Furthermore, situations involving complicated mission scenarios, numerous configuration changes, and many operational levels are not amenable to analytic solutions.

6.3 COMPARISON OF SIM3 AND GEMJR RESULTS

Sample runs of the configuration pictured in Figure 3 were made with SIM3 and GEMJR. The results are shown in Table 3. The average deviation of the results for all three phases was 1.4 percent.

TABLE 3 - SIM3 AND GEMJR RESULTS

	Reliability			Run Characteristics		
	Phase 1	Phase 2	Phase 3	System Seconds	Average Weighted Core	Estimated $ Cost
SIM3	0.926	0.746	0.590	31	19752	5.50
GEMJR	0.925	0.759	0.605	142	19074	21.24

Notes:
System Seconds = Computer central processing and input/output time.
Average Weighted Core = Average computer word memory in decimal units.
Estimated $ Cost = Projected cost of the run (based on system seconds).

The results in Table 3 show that the computer memory requirements of the two programs are similar. However, GEMJR required more than four times the computer time of SIM3 at nearly four times the cost. Thus when cost and time become factors in running the programs (especially with large reliability configurations), SIM3 will be preferred. However, before a final choice is made, other advantages and disadvantages should be reviewed.

APPENDIX A

EXAMPLE OF A TRADEOFF ANALYSIS USING SIM3

A hypothetical system is used to illustrate the integration of maintainability with optimization. The total system cost, consisting of both equipment and maintenance costs, will be optimized (i.e., minimized) by varying the configuration of the system.

1.0 OVERVIEW

In this hypothetical application SIM3 is used to generate equipment failures in the system. Each failure requires a corrective (or unscheduled) maintenance (CM) action, with a specific cost involved. Preventive (or scheduled) maintenance (PM) actions are generated at a fixed rate and the cost of each action is determined. The inclusion of PM effectively increases the MTBF of each equipment, decreasing the chance of failures (corrective maintenance requirements). There is a tradeoff between cost invested in PM and costs incurred in CM, for increased PM costs can decrease CM costs. Although not performed in this analysis, such a tradeoff could have been made to determine the minimum cost of both PM and CM actions.

2.0 METHOD

The following example illustrates the method of minimizing system costs. An optimization procedure is used to allocate equipment in the configuration.

Let C_M = maintenance cost of system

N_{A_i} = number of maintenance actions required for ith equipment

C_{A_i} = maintenance cost for ith equipment

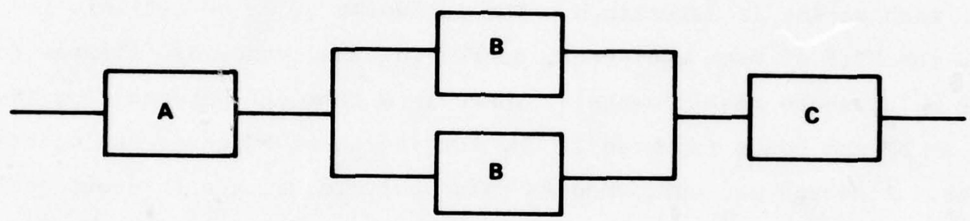
N = total number of equipments in the system

The expression for the maintenance costs of the system is

$$C_M = \sum_{i=1}^{N} N_{A_i} C_{A_i}$$

We can determine the effects on the maintenance cost of adding items in parallel or in series, and the effect of operating the system under different up-state rules, i.e., in a redundant configuration specifying how many equipments are required for the system to be in operation.

Let system X contain initially the different equipments A, B, and C



From this initial or baseline configuration we shall derive an optimal configuration for minimizing total cost by varying the number of equipments in the configuration.

Let C_A = cost of equipment A

C_B = cost of equipment B

C_C = cost of equipment C

C_M = total cost of maintenance actions

C_{MA} = cost of maintenance action on A

C_{MB} = cost of maintenance action on B

C_{MC} = cost of maintenance action on C

C_P = cost of preventive maintenance actions

C_{ST} = total system cost

n_A = number of items of A $\quad (n_A)_0 = 1$

n_B = number of items B $\quad (n_B)_0 = 2$

n_C = number of items C $\quad (n_C)_0 = 1$

$\left[\begin{array}{l} (n_i)_0 \text{ represents} \\ \text{the initial} \\ \text{configuration} \end{array}\right]$

N_A = number of maintenance actions on A

N_B = number of maintenance actions on B

N_C = number of maintenance actions on C

In the case of redundancies, the N's represent the total maintenance actions for all redundant items.

The cost of the equipment in system X is

$$C_S = n_A C_A + n_B C_B + n_C C_C$$

The maintenance cost due to repairs is

$$C_M = N_A C_{MA} + N_B C_{MB} + N_A C_{MC}$$

N_A, N_B, and N_C are obtained from the simulation program SIM3 and n_A, n_B, and n_C are the variables in the allocation optimization method. As indicated in the baseline configuration the initial values are

$$n_A = 1,\ n_B = 2,\ \text{and } n_C = 1$$

The objective is to minimize total system cost, C_{ST} by varying the n's where

$$C_{ST} = C_M + C_S + C_P$$

and C_P is the cost of preventive maintenance which occurs at a specified rate.

If R_A is the reliability of item A, R_B for B, and R_C for C, then the reliability without repair of the system is given by the expression

$$R = [1-(1-R_A)^{n_A}][1-(1-R_B)^{n_B}][1-(1-R_C)^{n_C}]$$

with the constraint $n_A, n_B, n_C \geq 1$.

This expression considers n_A redundancies for item A, n_B for B, and n_C for C. In the case of reliability with repair a closed expression with repair cannot be computed.

At this point, expressions for the total cost of the system, C_{ST}, and for the reliability, R, have been derived. The optimization process used when n_A, n_B, and n_C are varied will depend on whether the objective of the analysis is to

a. Minimize C_{ST}

b. Maximize R, or

c. Maximize R with a constraint on C_{ST}

In (a), where C_{ST} is minimized, the reliability of the system might become unacceptably low. In (b), as the R is maximized, the cost of the system can get unacceptably high. However, if R is required at a specific level regardless of cost, then (b) is used. Generally (c) provides a moderate approach to system design. Reliability is maximized within a specific cost constraint. Various values can be substituted for the cost constraint to obtain the variation of reliability with cost. If values of the reliability are too low within the range of acceptable costs, the designs of the system will have to be reevaluated.

To illustrate how the consideration of a weight constraint in the optimization of a configuration would be taken into account, let us return to our prior example.

Representing the total cost of system X as

$$C_{ST} = N_A C_A + N_B C_B + N_C C_C + N_A C_{MB} + N_B C_{MB} + N_C C_{MC}$$

with W_A, W_B, and W_C representing the weights of A, B, and C, and with W_X representing the weight of system X then

$$W_X = n_A W_Z + n_B W_B + n_C W_C$$

We then want to minimize C_{ST}, satisfying the weight constraint which is formulated as follows

$$W_X \leq W_0$$

where W_0 is a specified upper limit of the system weight. The system with the lowest C_{ST} for which the above inequality holds is desired.

APPENDIX B

SIM3 PROGRAM DESCRIPTION

1.0 INTRODUCTION

The simulation program SIM3 is a Fortran program originally developed by the Naval Applied Science Laboratory (NASL) for the CDC 3200/3300 series computers and converted for use on the CDC 6700 series.

Equipment failures and repairs are generated through a sequence of phases, i.e., a mission scenario. The same equipment can be used from phase to phase but the reliability configurations (block diagrams) can change.

DTNSRDC has added several features to the NASL version of SIM3:

- Printing option when the system aborts.
- Computation of total reliability for a mission.
- Computation of reliability for each phase.
- Tabulation of individual equipment failures for each phase.

2.0 INPUT DATA FORMAT AND DESCRIPTION

The input data pack consists of 13 different types of cards, some of which are repeated when the system consists of more than one phase. The first two cards contain indicators to control printing, as described in the output section. Cards 3 and 4, which describe equipment, are repeated as required. Card 5 indicates the end of the reliability data. Cards 6 and 7 describe the system and its phases and are repeated for each phase. A set of Cards 8 and 9 is required for each subsystem in each phase. Cards 10 and 11 are repeated as required to describe the block diagrams of each subsystem in each phase. Card 12 is used to renew equipment. For each run all the equipment must be renewed in the first phase. All values are right-justified. A blank card must be supplied if there is no input for one of the cards. There must also be a blank card between the last type card and the first equipment card. See Figure 4 for input deck setup. A description of the contents of each different type of card follows.

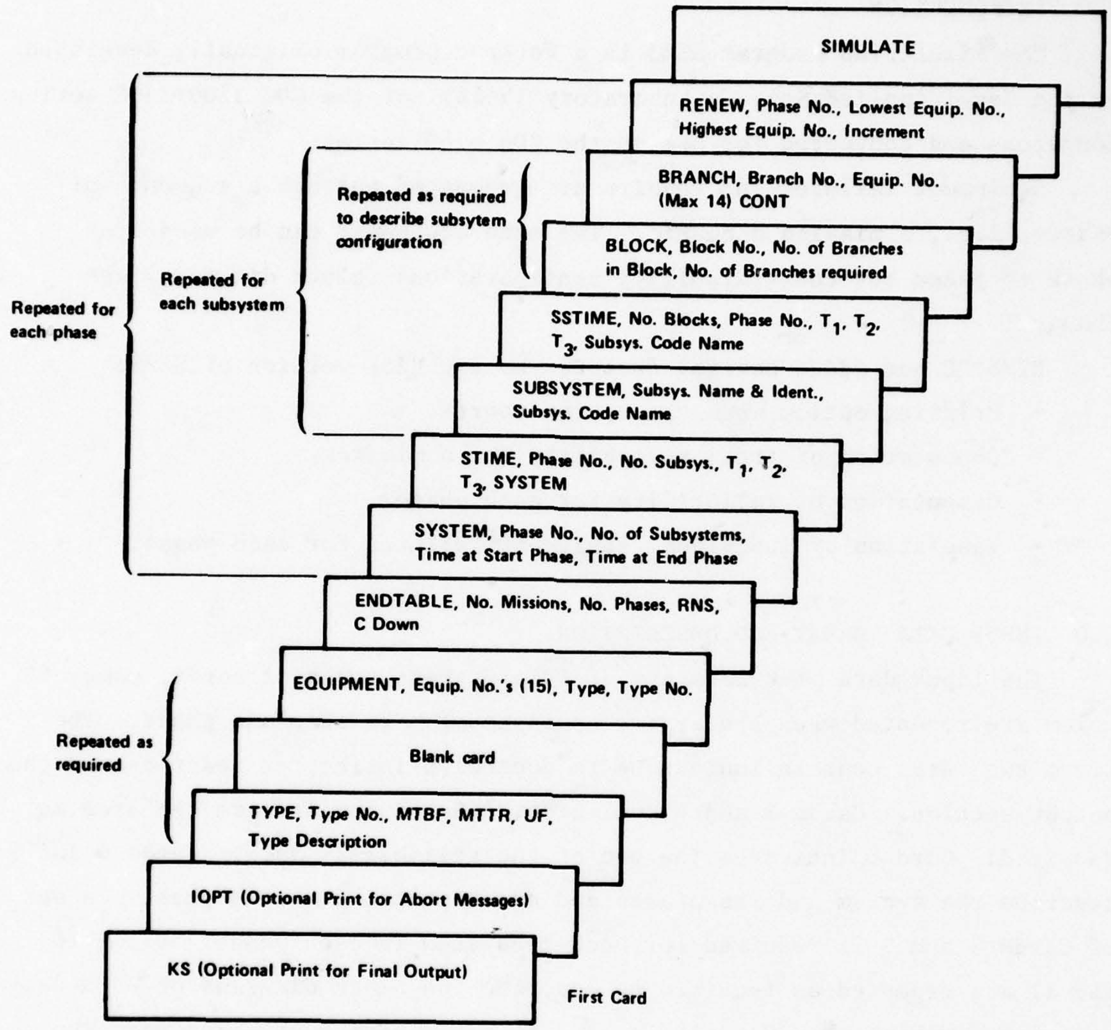


Figure 4 - SIM3 Source Deck Setup

	Variable Name	Cols.	Description
Card 1 - Indicator Card	KS(1)	10	Replace jump switches in
	KS(2)	20	original SIM3 routine.
	KS(3)	30	If KS (1 to 6) ≠ 2, all
	KS(4)	40	optional output will be
	KS(5)	50	printed. A value of 2 will
	KS(6)	60	suppress printout.
Card 2 - Indicator Card	IOPT	1-4	IOPT = 1 Print abort messages
			IOPT = 2 Omit abort messages
Card 3 - Type Card*	ID	2-5	The word TYPE punched
	I	11-14	Type number
	X	21-30	Mean time between failures (MTBF)
	Y	31-40	Mean time to repair (MTTR)
	U	41-50	Utilization factor
	F	73-80	Type name (Any alpha-numeric designation)
Card 4 - Equipment Card**	ID	2-5	The letters EQUI punched
	LOAD (1 to 15)	12-15 16-19 ⋮ 68-71	Equipment numbers, each consisting of four digits for a maximum of 15 different equipments on one card
	IQ	73-76	The word TYPE punched
	IT	77-80	Type number
Card 5 - End Table Card	ID	2-5	The letters ENTA punched
	LOAD(1) LOAD(2)	12-19	Number of missions to be run
	LOAD(4)	24-27	Number of phases to be run
	LOAD(6)	32-35	Initial random number (Not used at present)
	LOAD(8)	40-43	C Down. If number of system failures > C Down, sum system downtimes on a separate counter
Card 6 - System Card	ID	2-5	The letters SYST punched
	JPHASE	11-14	Phase number
	NSS	17-19	Number of subsystems in system

---

*Card 3 is repeated for each type number in the configuration.

**Card 4 is repeated for each piece of equipment.

	Variable Name	Cols.	Description
Card 6 (Continued)	STPHAS	21-30	Calendar time at start of phase
	ENDPHA	31-40	Calendar time at end of phase
Card 7 - System Time Card	ID	2-5	The letters STIM punched
	JPHASE	11-14	Phase number
	NSS	17-19	Number of subsystems in the system
	SSTIME(1)	21-30	System allowable negligible downtime (T_1)
	SSTIME(2)	31-40	System allowable sustained downtime (T_2)
	SSTIME(3)	41-50	System allowable cumulative downtime (T_3)
	F	73-80	The word SYSTEM punched
Card 8 - Subsystem Card	LOAD(1)	2-5	The letters SUBS punched
	LOAD(2)	6-9	
	⋮	⋮	
	LOAD(19)	74-77	Subsystem name and identification
	LOAD(20)	78-80	Subsystem code name, any designation desired
Card 9 - Subsystem Time Card	ID	2-5	The letters SSTI punched
	MBL	11-14	Number of blocks in the subsystem
	K	17-19	Phase number
	SSTIME(1)	21-30	Subsystem allowable negligible downtime (T_1)
	SSTIME(2)	31-40	Subsystem allowable sustained downtime (T_2)
	SSTIME(3)	41-50	Subsystem allowable cumulative downtime (T_3)
	TITLE	73-80	Subsystem code name
Card 10 - Block Card*	ID	2-5	The letters BLOC punched
	IBL	12-15	Block number
	MBR	16-19	Number of branches in block
	NEED	20-23	Number of branches required for operation

*Card 10 is repeated to describe the entire subsystem configuration.

	Variable Name	Cols.	Description
Card 11 - Branch Card*	ID	2-5	The letters BRAN punched
	IBR	12-15	Branch number
	LOAD(1)	16-19	Equipment numbers in branch (4 cols. for each equipment no.; max. of 14 equipments)
	LOAD(2)	20-23	
	⋮	⋮	
	LOAD(14)	68-71	
	IKK	73-76	The word CONT is punched in cols. 73-76 if there are more than 14 equipments in the Branch. Additional equipment nos. start in column 16 of the continuing card(s).
Card 12 - Renew Card	ID	2-5	The letters RENE punched
	IQ	12-15	Number of phase in which renew action is to take place
	KL	16-19	Lowest equipment number in the set
	KH	20-23	Highest equipment number in the set
	INC	24-27	Increment counter for renewal of equipment INC = 1 (or blank), renew each equipment; INC = 2, renew every other one; INC = 3, renew every third equipment

Note: Equipment to be renewed at the start of any phase must be grouped in sets of consecutive numbers.

Example: To renew the equipments 10 to 20 and 50 to 75, two different RENEW cards are required.

Card 13	ID	2-5	The letters SIMU punched

3.0 OUTPUT

SIM3 output falls into two categories: automatic and optional. Optional output is generated by the program when the elements of KS are not equal to 2. Automatic output, printed only at certain times, includes

*Card 11 is repeated to describe the entire subsystem configuration.

• Printout of all input cards.

• Printout of mission abort data including the equipments that caused the abort, the time of abort, and the time that the equipment will come up again.

• Summary tabulation at the end of each phase, containing the number of simulated missions that entered the phase and the number aborted during the phase.

The optional output is controlled by the values assigned to the KS array as follows:

KS(1) = 0	Print out equipment state table at time of abort
KS(2) = 0	Print out each equipment state table if system is down at end of phase
KS(3) = 0	Each time system goes down, print out the time that it went down, how long it stayed down, and the time it resumed operation
KS(4)	Not used in this version
KS(5) = 0	Print out each event as it occurs during the simulation
KS(6) = 0	Internal tables of events occurring during simulation This is a debugging feature.

A value of 2 for any of the above will prevent printing.

4.0 PROGRAM LISTING

```
      PROGRAM MIKE(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE1,TAPE2)
      DIMENSION KS(10),F(2)
      DIMENSION TI2(50)
      DIMENSION LOAD(20)  ,REL(10)
      DIMENSION IWORD(6), ITYPE(20), NFAILS(1500), KEQU(1500)
      EQUIVALENCE (ETIME(1),IWORD(1))
      COMMON XMTBF(500),XMTTR(500),IDOWN(800),LBLOC1 (250),LBLOC2 (250)
     1,TITLE(50),ISUB1(25),ISUB2(25),SSTIME(5,25),LOST1(25),LOST2(25)
     2,LOST3(25)
      EQUIVALENCE  (LOAD,IDOWN)
      COMMON  IEQU(1500),ETIME(1500)
      COMMON STPHAS ,ENDPHA  ,NEQ,JPHASE,TP,NPH,NMI,NSMISS,NBR,NBL,NSS,
     1NSS1,ISWBL,ISWSS
      DATA JTYPE,JENDT,JENDS/4HTYPE,4HENDT,4HENDS/
      DATA JCONT,JRENE/4HCONT,4HRENE/
C
C     IOPT=1-------------------PRINT ABORT MESSAGES
C     IOPT=2-------------------OMIT ABORT MESSAGES
      READ (5,606) (KS(I),I=1,6)
      READ (5,607) IOPT
  606 FORMAT (8I10)
  607 FORMAT (I4)
C
C
C  FILL  EQU AND  TYPE  TABLES
      IREAD=1
      IWRITE=2
      NEQ=0
      NTYPE=0
      NBL=0
      NBR=0
      NSS=0
      TOTALR = 1.0
      RELP = 1.0
      IDATA = 1
      DO 1 I = 1,250
      LBLOC1(I) = 0
    1 LBLOC2(I) = 0
      DO 2009  I=1,1500
      ETIME(I) = 1.E30
      IEQU(I)=0
      NFAILS(I) = 0
      KEQU(I) = 0
 2009 CONTINUE
      DO 2408 I =1,500
      XMTBF(I)=0.0
 2408 XMTTR(I)=0.0
C READ  TYPE CARDS
 2010 READ(5,2001)ID,I,K,X,Y,U,(F(J),J=1,2)
 2001 FORMAT(1X,A4,5X,I4,2X,I3,1X,3E10.3,22X,2A4)
      IF(ID.NE.JTYPE) GO TO 2012
 2011 WRITE(6,2001)ID,I,K,X,Y,U,(F(J),J=1,2)
      IF(XMTBF(I))2051,2050,2051
 2051 WRITE(6,2055)I
      GO TO 2010
 2055 FORMAT(1X,4HTYPE,I5,1X,13HDEFINED TWICE)
 2050 XMTBF(I)=X/U
```

```
      XMTTR(I)=Y
      NTYPE=NTYPE+1
      GO TO 2010
C AFTER  LAST TYPE CARD  MUST BE A BLANK CARD .THEN FOLLOWS EQU CARDS
 2012 READ(5,2002)ID,(LOAD(I),I=1,15),IQ,IT
      IF(ID.EQ.JENDT) GO TO 2013
 2014 DO  2015 I=1,15
      IF(LOAD(I).EQ.0) GO TO 2015
 2016 IBM=LOAD(I)
      IF(IBM.LE.NEQ)  GO TO 2096
 2095 NEQ = IBM
 2096 IF(INPAKH  (IEQU(IBM)))2061,2069,2061
 2061 WRITE(6,2065)IBM
      GO TO 2012
 2065 FORMAT(1X,4HEQU.,I5,1X,13HDEFINED TWICE)
 2069 CONTINUE
      IEQU(IBM)=IPACKH(IT,IEQU(IBM))
 2015 CONTINUE
      WRITE(6,2002)ID,(LOAD(I),I=1,15),IQ,IT
      GO TO 2012
C
C  ALL  EQV AND TYPE  CARDS HAVE BEEN READ IN THE LAST CARD READ AT
C     THIS POINT  WAS AN ENDTABLE  CARD
 2013 NMI=LOAD(1)*10+LOAD(2)
      NSMISS=NMI
      CDOWN =LOAD(8)
      NPH=LOAD(4)
      LAND=LOAD(6)
C     CALL RANSET(LAND,Y)
      WRITE(6,2004)NMI,NPH
 2004 FORMAT(1H1,5X,I8,1X,25HMISSIONS WILL BE RUN THRU,I4,1X,7HPHASES.)
C
C
C PHASE  GEOMETRY CARDS  SHOULD APPEAR NEXT  OR ENDSIM
C     NEXT CARD  SHOULD  BE  A SYSTEM CARD
 2100 WRITE(6,2004)
      DO 2401 I = 1,NEQ
 2401 IEQU(I)=AND(IEQU(I),4000000000007777000OB)
      NMISS=NSMISS
      WRITE(6,999)IREAD
  999 FORMAT(3H LU,I3,1X,19HINPUT TO NEXT PHASE)
      READ(5,2001)ID,JPHASE,NSS,STPHAS ,ENDPHA
      IF(ID.NE.JENDS) GO TO 2121
C2020 IS  AFTER LAST  PHASE  IT CALLS  FOR FINAL SUMMARY TABLES
C2020 CALL UNLOAD(IWRITE)
C     COMPUTE TOTAL RELIABILITY   FOR COMPLETE MISSION**************************
      DO 1190 JJ = 1, NPH
      TOTALR = REL(JJ) * TOTALR
 1190 CONTINUE
      WRITE(6, 895) TOTALR, NPH
  895 FORMAT (1X, 40HTOTAL RELIABILITY FOR COMPLETE MISSION= , F8.5,
     114HAT THE END OF , I4, 9H PHASES  )
      STOP
 2121 WRITE(6,2001)ID,JPHASE,NSS,STPHAS ,ENDPHA
      NSS1=NSS+1
      READ(5,2001)ID,JPHASE,NSS,(SSTIME(M,NSS1),M=1,3),F
      WRITE(6,2001)ID,JPHASE,NSS,(SSTIME(M,NSS1),M=1,3),F
```

```
      TITLE(NSS1)=F(1)
      TI2(NSS1) = F(2)
      DO 200 I=1,NSS1
      LOST1(I)=0
      LOST2(I)=0
      LOST3(I)=0
      ISUB1(I)=0
  200 ISUB2(I)=0
C  DO FOR  ALL  SUBSYSTEMS
      LBL=1
      LBR=1
C  LBL IS LOW BLOCK NUMBER  IN THIS SS.
C  LBR IS LOW BRANCH  NUMBER  IN THIS BLOCK.
      DO  2200 II=1,NSS
      READ(5,2003)LOAD
 2003 FORMAT(1X,19A4,A3)
      WRITE(6,2003)LOAD
      READ(5,2001) ID,MBL,K,(SSTIME(M,II),M=1,3),TITLE(II),TI2(II)
      WRITE(6,2001)ID,MBL,K,(SSTIME(M,II),M=1,3),TITLE(II),TI2(II)
      ISUB1(II)=LBL
      ISUB2(II)=LBL+MBL-1
C
C DO FOR ALL  BLOCKS IN THIS SUBSYSTEM
      DO 2202   JJ=1,MBL
      READ(5,2002) ID,IBL,MBR,NEED
      WRITE(6,2002)ID,IBL,MBR,NEED
      MQ=IBL+LBL -1
      LBLOC1 (MQ) = IPACKL(LBR,        LBLOC1 (MQ))
      LBLOC1 (MQ) = IPACKH(LBR+MBR-1,   LBLOC1 (MQ))
      LBLOC2 (MQ) = IPACKH(II ,         LBLOC2 (MQ))
      LBLOC2 (MQ) = IPACKL(NEED,       LBLOC2 (MQ))
C
C DO FOR ALL  BRANCH CARDS IN THIS BLOCK
      DO  2201  KK=1,MBR
 2110 READ(5,2002) ID,IBR,(LOAD(N),N=1,14),IKK
      WRITE(6,2002)ID,IBR,(LOAD(N),N=1,14),IKK
 2002 FORMAT(1X,A4,6X,15I4,1X,A4,I4)
      DO 2115 N=1,14
      IT=LOAD(N)
      IF(IT.EQ.0) GO TO 2115
 2116 IQ=IBR+LBR-1
      IF (INPAKL  (IEQU(IT)))2071,2070,2071
 2071 WRITE(6,2075)IT
      IK = KS(4)
      IK = 1
      IF(IK.EQ.2) GO TO 2110
C2500 CALL UNLOAD(IWRITE)
 2500 CONTINUE
 2075 FORMAT(1X,4HEQU.,I5,1X,23HAPPEARS IN TWO BRANCHES)
 2070 IEQU(IT)=IPACKL(IQ,IEQU(IT))
 2115 CONTINUE
      IF(IKK.EQ.JCONT) GO TO 2110
 2201 CONTINUE
      LBR =LBR +MBR
 2202 CONTINUE
      LBL =LBL +MBL
 2200 CONTINUE
```

```
      NBL=LBL-1
      NBR=LBR-1
      ISUB1(NSS1)=1
      ISUB2(NSS1)=NBL
C
C  PHASE  GEOMETRY  HAS BEEN READ IN
C
C
C  NEXT  CARD  SHOULD  BE A RENEW  CARD  OR A SIMULATE CARD
C   FIRST  CLEAR  RENEWS  FROM  PREVIOUS  PHASES
      DO   2400   I=1,NEQ
 2400 IEQU(I)=AND(00000000000037777777B,IEQU(I))
 2156 READ(5,2002)ID,IQ,KL,KH,INC
      IF(ID.NE.JRENE) GO TO 2151
 2150 IF(INC)2152,2153,2152
 2153 INC=1
 2152 DO  2155  K=KL,KH,INC
 2155 IEQU(K)=OR(40000000000000000000B,IEQU(K))
      WRITE(6,2002)ID,IQ,KL,KH,INC
      GO TO 2156
C
C SIMULATE CARD       START SIMULATION
C START OF PHASE  JPHASE
 2151 DO 401     KMI=1,NMI
      TIME=STPHAS
      SUMDOW =0.0
      SUNDOW =0.0
      IF (JPHASE-1)     10,19,18
   10 READ(IREAD)        ITEMP,ITEMP1,ITEMP2,TEMP1,TEM12,TEMP2,
     1  (ETIME(J),J = 1,NEQ)
C IF FIRST WORD=0,MISSION WAS SUCCESS
      IF (ITEMP)     18,19,18
   18 NMISS=NMISS-1
      GO TO 300
   19 DO  20     K=1,NBR
   20 IDOWN(K)=0
C RENEW EQUIPMENTS SPECIFIED AND DETERMINE INITIAL STATE OF BRANCHES
      DO  25     I=1,NEQ
      IF (IEQU(I))    921,25,22
  921 IF(INPAKH   (IEQU(I)))21,25,21
   21 CALL SETE(1,I)
      GO TO 25
   22 IF (ETIME(I))     24,23,25
C ETIME=0 BUT IEQU NOT =0
   23 WRITE(6,610)
  610 FORMAT(1H0,5X,6HETIME(,I2,5H) = 0,5X,4HMAIN)
   24 K=INPAKL   (IEQU(I))
      IDOWN(K)=IDOWN(K)+1
   25 CONTINUE
C SET INITIAL CONDITIONS OF BLOCKS,SUBSYSTEMS AND SYSTEM
      ISUB1(NSS1)=AND(00000000000037777777B,ISUB1(NSS1))
      DO  50     KSS=1,NSS
      CALL SSUP(1,KSS)
      IF (ISWSS)     45,46,45
   45 ISUB1(NSS1)=OR(40000000000000000000B,ISUB1(NSS1))
   46 SSTIME(4,KSS)=0.
      SSTIME(5,KSS)=0.
```

```
   50 CONTINUE
      SSTIME(4,NSS1)=0.
      SSTIME(5,NSS1)=0.
C THE ACTUAL MISSION SIMULATION BEGINS HERE
   60 TP=TIME
      IF(KS(6)-2)51,54,51
   51 WRITE(6,52)TP,(IEQU(J),ETIME(J),J=1,NEQ)
      WRITE(6,53)(IDOWN(K),K=1,NBR),(LBLOC2 (I),I=1,NBL),(ISUB1(J),J=1,N
     1SS1),(ISUB2(JJ),JJ=1,NSS1)
   52 FORMAT(1X,F10.4,/(O9,F10.4))
   53 FORMAT(1X,O8)
   54 CALL EVENT(TIME,IFORR,KEQ)
      IK = KS(5)
      IF(IK.EQ.2) GO TO 92
   91 WRITE(6,90)KEQ,ETIME(KEQ),KMI
   90 FORMAT(I20,F10.3,5X,7HMISSION,I10)
   92 DELT=TIME-TP
C CHECK IF ANY DOWN TIMES HAVE EXCEEDED CRITERIA
      DO  70   KSS=1,NSS1
      IF (ISUB1(KSS))     65,64,70
C ISUB1=0 FOR SOME SUBSYSTEM OR SYSTEM
   64 WRITE(6,601) KSS
  601 FORMAT(1H0,5X,6HISUB1(,I2,5H) = 0)
   65 SSTIME(4,KSS)=SSTIME(4,KSS)+DELT
      IF(SSTIME(4,KSS)-SSTIME(2,KSS))   66,66,202
   66 IF (SSTIME(4,KSS)-SSTIME(1,KSS))    70,70,67
   67 IF (SSTIME(5,KSS)+SSTIME(4,KSS)-SSTIME(3,KSS))     70,70,203
   70 CONTINUE
   71 CONTINUE
C CHECK IF TIME GREATER THAN END OF PHASE
      IF (TIME-ENDPHA  )     75,75,250
   75 KBR=INPAKL  (IEQU(KEQ))
C CHECK IF EQUIPMENT IS IN SYSTEM
      IF (KBR)     76,600,76
  600 CALL SETE(0,KEQ)
      GO TO 60
C FIND BLOCK WHICH EQUIPMENT IS IN
   76 DO  80     KBL=1,NBL
      IF (KBR-INPAKH  (LBLOC1 (KBL)))     85,85,80
   80 CONTINUE
C BRANCH NUMBER HIGHER THAN HIGHEST BRANCH IN HIGHEST BLOCK
      WRITE(6,602)
  602 FORMAT(1H0,5X,21H SEE COMMENT ABOVE 80)
   85 KSS=INPAKH  (LBLOC2 (KBL))
      ISSPRE =ISUB1(KSS)
      ISYPRE =ISUB1(NSS1)
C PERFORM EVENT AND UPDATE STATE OF BRKBR,BLKBL,SBSYSTEM KSS AND SYSTEM
      CALL SETE (9,KEQ)
      IDOWN(KBR)=IDOWN(KBR)+IFORR
      CALL BLOCUP(KBL)
      IF (IFORR)     120,100,101
C EVENT NEITHER FAILURE NOR REPAIR, OR STATE WAS NEITHER UP OR DOWN
  100 WRITE(6,603)
  603 FORMAT(1H0,5X,22H SEE COMMENT ABOVE 100)
C EVENT WAS FAILURE
  101 IF (ISWBL)     110,60,110
  110 IF (ISSPRE )     60,100,112
```

```
  112 ISUB1(KSS)=OR(40000000000000000000B,ISUB1(KSS))
      IF (ISYPRE )    60,100,114
  114 ISUB1(NSS1)=OR(40000000000000000000B,ISUB1(NSS1))
      GO TO 60
C EVENT WAS REPAIR
  120 IF (IDOWN (KBR))    60,122,60
  122 IF (ISWBL)    60,125,60
  125 CALL SSUP(0,KSS)
      IF ( ISWSS)    60,126,60
  126 IF (ISSPRE )    127,100,60
  127 LOST1(KSS)=LOST1(KSS)+1
      IF (SSTIME(4,KSS)-SSTIME(1,KSS))     140,140,130
  130 SSTIME(5,KSS)=SSTIME(5,KSS)+SSTIME(4,KSS)
  140 SSTIME(4,KSS)=0.
      CALL SYSUP
      IF (ISUB1(NSS1))    60,100,154
  154 IF (ISYPRE )    155,100,60
  155 LOST1(NSS1)=LOST1(NSS1)+1
      XQX=SSTIME(4,NSS1)
      TDOWN=TIME-XQX
      SUMDOW =SUMDOW +XQX
      IF(XQX.LE.CDOWN) GO TO 451
  450 SUNDOW  = SUNDOW  + XQX
  451 IF(KS(3)-2) 156,165,156
  156 WRITE(6,226)JPHASE,TDOWN,TIME,SSTIME(4,NSS1),KMI
  165 IF(SSTIME(4,NSS1)-SSTIME(1,NSS1))     180,180,170
  170 SSTIME(5,NSS1)=SSTIME(5,NSS1)+SSTIME(4,NSS1)
  180 SSTIME(4,NSS1)=0.
      GO TO 60
C ABORT PROCEDURE
  202 ICRIT=2
      TABORT=TIME-(SSTIME(4,KSS)-SSTIME(2,KSS))
      GO TO 204
  203 ICRIT=3
      TABORT=TIME-(SSTIME(4,KSS)+SSTIME(5,KSS)-SSTIME(3,KSS))
  204 IF (TABORT-ENDPHA  )    205,71,71
  205 IF (ICRIT-2)    206,207,206
  206 LOST3(KSS)=LOST3(KSS)+1
  207 LOST2(KSS)=LOST2(KSS)+1
      IF(IOPT.EQ.2) GO TO 209
  208 WRITE(6,220)JPHASE,KMI,TABORT,TITLE(KSS),TI2(KSS),ICRIT,SSTIME
     1   (ICRIT,KSS)
  209 IF(IDATA.EQ.2)GO TO 215
  210 JKMI = 0
      IDATA = 2
  215 IJ = 1
      DO 516 I =1,NEQ
      ITYPE(IJ) = I
      IJ = IJ + 1
      IF(ETIME(I))513,516,516
  513 IF(INPAKH   (IEQU(I)))514,516,514
C     COMPUTE TOATL TIMES OF EACH EQUIPMENT FAILURE ***************************
  514 IF (KMI .EQ. JKMI) GO TO 512
      JKMI = KMI
      KEQU(I) = KEQU (I) + 1
      IF(IOPT.EQ.2) GO TO 516
  512 WRITE(6,515)I,ETIME(I)
```

```
  515 FORMAT(17X,9HEQUIPMENT,I5,24H DOWN IT WILL COME UP AT,F10.4)
  516 CONTINUE
      IK = KS(1)
      IF(IK.EQ.2) GO TO 518
  517 CALL PPROG
  518 ITEMP =ICRIT
      ITEMP1=KMI
      ITEMP2=JPHASE
      TEMP1=TITLE(KSS)
      TEM12 = TI2(KSS)
      TEMP2=TABORT
      GO TO 280
C END OF PHASE PROCEDURE FOR MISSION KMI
  250 IF(ISUB1(NSS1))     260,251,272
C SYSTEM NEITHER UP NOR DOWN, DITTO SUBSYSTEM
  251 WRITE(6,604)
  604 FORMAT(1H0,5X,17HSEE COMMENT ABOVE)
  260 TDOWN=TIME-SSTIME(4,NSS1)
      TDUR=ENDPHA  -TDOWN
      IF(KS(3)-2) 265,270,265
  265 WRITE(6,226)JPHASE,TDOWN,ENDPHA  ,TDUR,KMI
  270 WRITE(6,225)JPHASE,TDUR,KMI
      IF(KS(2)-2) 271,272,271
  271 CALL PPROG
  272 ITEMP=0
  280 DO 290     I=1,NSS1
      IF (ISUB1(I))      282,251,290
  282 LOST1(I)=LOST1(I)+1
  290 CONTINUE
  300 WRITE (IWRITE)    ITEMP,ITEMP1,ITEMP2,TEMP1,TEM12,TEMP2,
     1  (ETIME(J),J = 1,NEQ)
      IK = KS(3)
      IF(IK.EQ.2) GO TO 401
  400 IF(SUMDOW.EQ.0.0) GO TO 401
  410 WRITE(6,402)JPHASE,KMI,SUMDOW ,CDOWN,SUNDOW
  402 FORMAT(1X,5HPHASE,I5,1X,29HTOTAL SYS DOWNTIME IN MISSION,I5,1X,3HW
     1AS,E14.4,4H HRS/10X,14HSYS DOWNS .GT.,F6.0,8H HRS WAS,F10.4,4H HRS
     2)
  401 CONTINUE
C END OF PHASE JPHASE PROCEDURE
      WRITE(6,227)NMISS,JPHASE
C     WRITE EQUIPMENT NO. AND TOTAL FAILURES**********************************
      IDIFF = 0
      DO 1110 IJ = 1, NEQ
      WRITE (6,820) ITYPE(IJ), KEQU(IJ)
  820 FORMAT (1X, 15HEQUIPMENT NO.  , I4, 5X, 16HTOTAL FAILURES= ,I4)
      IDIFF = IDIFF + KEQU(IJ)
 1110 CONTINUE
      WRITE (6,821) IDIFF
  821 FORMAT (41X, 4H----, / 41X, I4)
      DO 1115 IJ = 1, NEQ
      KEQU(IJ) = 0
 1115 CONTINUE
      IDATA = 1
      DO 311     I=1,NSS1
      ITEMP=LOST2(I)+LOST3(I)
      RELL1 = LOST1(I)
```

```
  311 WRITE(6,228) TITLE(I),TI2(I),                    SSTIME(1,I),
     1SSTIME(2,I),SSTIME(3,I),LOST2(I),LOST3(I),ITEMP,LOST1(I)
C     COMPUTE RELIABILITY FOR EACH PHASE ****************************************
      RELNM = NMISS
      REL(JPHASE) = (RELNM - RELL1) / RELNM
      RELP = RELP * REL(JPHASE)
      WRITE (6, 800) JPHASE, REL(JPHASE), JPHASE, RELP

  800 FORMAT (/1X, 26HRELIABILITY DURING PHASE  ,I2, 3H = ,F5.3,
     1 13X, 25HRELIABILITY UP TO PHASE  ,I2, 3H = , F5.3)
      ITEMP=IWRITE
      ENDFILE IWRITE
      REWIND IWRITE
      REWIND IREAD
      IWRITE=IREAD
      IREAD=ITEMP
      GO TO 2100
  220 FORMAT(1X,9HIN PHASE ,I6,4X,8HMISSION ,I6,4X,15HABORTED AT TIME,F1
     10.4,10H  BECAUSE ,2A4,15H  EXCEEDED CRIT,I3,5X,F10.3,5H HRS.)
  225 FORMAT(1X,27HSYSTEM DOWN AT END OF PHASE,I6,13H FOR DURATION,F10.4
     1,6X,7HMISSION,I6)
  226 FORMAT(1X,12HDURING PHASE,I6,20H SYSTEM WENT DOWN AT,F10.4,18H SYS
     1TEM CAME UP AT,F10.4,11H DOWNTIME =,F10.4,6X,7HMISSION,I6)
  227 FORMAT(1X,I6,23H MISSIONS ENTERED PHASE,I6)
  228 FORMAT(1X,2A4,4X,3HT1=,F10.4,4X,3HT2=,F10.4,4X,3HT3=,F10.4,4X,7HAB
     1ORT2=,I6,4X,7HABORT3=,I6,4X,9HTOTABORT=,I6,4X,8HTOTDOWN=,I6)
      END



      SUBROUTINE  PPROG
      DIMENSION LOAD(20)
      DIMENSION  IWORD(6)
      EQUIVALENCE (ETIME(1),IWORD(1))
      COMMON XMTBF(500),XMTTR(500),IDOWN(800),LBLOC1 (250),LBLOC2 (250)
     1,TITLE(50),ISUB1(25),ISUB2(25),SSTIME(5,25),LOST1(25),LOST2(25)
     2,LOST3(25)
      EQUIVALENCE  (LOAD,IDOWN)
      COMMON  IEQU(1500),ETIME(1500)
      COMMON STPHAS ,ENDPHA  ,NEQ,JPHASE,TP,NPH,NMI,NSMISS,NBR,NBL,NSS,
     1NSS1,ISWBL,ISWSS
C
C
C
      DIMENSION IC(10)
      DIMENSION IJK(10)
      DATA J1HX,J1H/1HX,1H /
      DO 601 K=1,10
  601 IJK(K)=K-1
      WRITE(6,10) ENDPHA,(IJK(K),K=1,10)
   10 FORMAT(1X,13HPHASE ENDS AT,F12.3/40X,50HTABLE  OF EQUIPMENT STATES
     1  AND TIME OF NEXT EVENT/4H  NO,9X,I1,9(12X,I1)/)
      INEQ=NEQ+1
      DO 425 I=1,INEQ,10
      IJ=I-1
      L=IJ+9
      WRITE(6,11) IJ,(ETIME(J),J=IJ,L)
   11 FORMAT(1H ,I4,10(2X,F10.2))
      DO 2 J=1,10
      L=IJ+J-1
      IF (INPAKL  (IEQU(L)))41,40,41
   40 IC(J)=J1HX
      GO TO 2
   41 IC(J)=J1H
    2 CONTINUE
  425 WRITE(6,12)IC
   12 FORMAT(1X,10(12X,A1))
      RETURN
      END
```

```
      SUBROUTINE    SETE(KEY,N)
      DIMENSION LOAD(20)
      DIMENSION  IWORD(6)
      EQUIVALENCE (ETIME(1),IWORD(1))
      COMMON XMTBF(500),XMTTR(500),IDOWN(800),LBLOC1 (250),LBLOC2 (250)
     1,TITLE(50),ISUB1(25),ISUB2(25),SSTIME(5,25),LOST1(25),LOST2(25)
     2,LOST3(25)
      EQUIVALENCE  (LOAD,IDOWN)
      COMMON  IEQU(1500),ETIME(1500)
      COMMON STPHAS ,ENDPHA  ,NEQ,JPHASE,TP,NPH,NMI,NSMISS,NBR,NBL,NSS,
     1NSS1,ISWBL,ISWSS
C
C
C
C  KEY  =1   GENERATE   TIME  TO FAILURE
C  KEY  =0   GENERATE   TIME  TO NEXT EVENT
C
      RN = RANF(DUM)
      ITYPE=INPAKH  (IEQU(N))
      IKEY=KEY+1
      IF(IKEY.EQ.2) GO TO 2
    1 IF(ETIME(N))3,4,5
    4 WRITE(6,10)N
   10 FORMAT(1H0,5X,6HETIME(,I2,5H) = 0,5X,4HSETE)
C  FIND   REPAIR  TIME
    5 ETIME(N)=-1.0*(-XMTTR(ITYPE)*ALOG(RN)+ETIME(N))
      RETURN
C    GENTERATE TIME  TO FAIL  IF KEY=0 RECKON TIME FROM START OF PHASE
    2 B=STPHAS
      GO TO 6
    3 B=ABS(ETIME(N))
    6 ETIME(N)=-XMTBF(ITYPE)* ALOG(RN)+ B
      RETURN
      END
```

```
      FUNCTION IPACK(I,K)
      ENTRY IPACKH
      IF(I.GT.2047) GO TO 3
      IPACK=I*4096 + AND(00000000000000077778,K)
      RETURN
      ENTRY IPACKL
      IF(I.GT.2047) GO TO 3
      IPACK = I + AND(00000000000037770000B,K)
      RETURN
    3 WRITE(6,5)
    5 FORMAT(1H0,5X,9HI.GT.2047)
      RETURN
      END
```

```
      FUNCTION INPAK(I)
      ENTRY INPAKL
      INPAK = AND(00000000000000077773,I)
      RETURN
      ENTRY INPAKH
      INPAK = AND(00000000000037770000B,I)/4096
      RETURN
      END
```

```
      SUBROUTINE EVENT(TIME,LSIGN,LPOS)
      DIMENSION LOAD(20)
      DIMENSION  IWORD(6)
      EQUIVALENCE (ETIME(1),IWORD(1))
      COMMON XMTBF(500),XMTTR(500),IDOWN(800),LBLOC1 (250),LBLOC2 (250)
     1,TITLE(50),ISUB1(25),ISUB2(25),SSTIME(5,25),LOST1(25),LOST2(25)
     2,LOST3(25)
      EQUIVALENCE  (LOAD,IDOWN)
      COMMON  IEQU(1500),ETIME(1500)
      COMMON STPHAS ,ENDPHA  ,NEQ,JPHASE,TP,NPH,NMI,NSMISS,NBR,NBL,NSS,
     1NSS1,ISWBL,ISWSS
C
C
C
      TIME=ABS(ETIME(1))
      LPOS=1
      DO 2 J=1,NEQ
      R=ABS(ETIME(J))
      IF(R-TIME)11,2,2
   11 TIME=R
      LPOS=J
    2 CONTINUE
      IF(ETIME(LPOS))3,4,4
    3 LSIGN=-1
      GO TO 5
    4 LSIGN=1
    5 RETURN
      END
```

```
      SUBROUTINE  SYSUP
      DIMENSION LOAD(20)
      DIMENSION  IWORD(6)
      EQUIVALENCE (ETIME(1),IWORD(1))
      COMMON XMTBF(500),XMTTR(500),IDOWN(800),LBLOC1 (250),LBLOC2 (250)
     1,TITLE(50),ISUB1(25),ISUB2(25),SSTIME(5,25),LOST1(25),LOST2(25)
     2,LOST3(25)
      EQUIVALENCE  (LOAD,IDOWN)
      COMMON  IEQU(1500),ETIME(1500)
      COMMON STPHAS ,ENDPHA  ,NEQ,JPHASE,TP,NPH,NMI,NSMISS,NBR,NBL,NSS,
     1NSS1,ISWBL,ISWSS
C
C
C
      DO 70    I=1,NSS
      IF (ISUB1(I))     62,61,70
C SUBSYSTEM IS NEITHER UP NOR DOWN
   61 WRITE(6,5)
    5 FORMAT(1H0,5X,17HSEE COMMENT ABOVE,5X,5HSYSUP)
   62 ISUB1(NSS1)=OR(40000000000000000000B,ISUB1(NSS1))
      GO TO 75
   70 CONTINUE
      ISUB1(NSS1)=AND(00000000000377777777B,ISUB1(NSS1))
   75 RETURN
      END
```

```
      SUBROUTINE  SSUP(KKK,K)
      DIMENSION LOAD(20)
      DIMENSION  IWORD(6)
      EQUIVALENCE (ETIME(1),IWORD(1))
      COMMON XMTBF(500),XMTTR(500),IDOWN(800),LBLOC1 (250),LBLOC2 (250)
     1,TITLE(50),ISUB1(25),ISUB2(25),SSTIME(5,25),LOST1(25),LOST2(25)
     2,LOST3(25)
      EQUIVALENCE  (LOAD,IDOWN)
      COMMON  IEQU(1500),ETIME(1500;
      COMMON STPHAS ,ENDPHA  ,NEQ,JPHASE,TP,NPH,NMI,NSMISS,NBR,NBL,NSS,
     1NSS1,ISWBL,ISWSS
C
C
      ISWSS=0
      ISUB1(K)=AND(00000000000037777777B,ISUB1(K))
      ILOBL=ISUB1(K)
      IHIBL=ISUB2(K)
      DO 50    J=ILOBL,IHIBL
      IF (KKK)   41,42,41
   41 CALL BLOCUP(J)
   42 IF(LBLOC2 (J))    43,45,50
   43 ISWSS=1
      ISUB1(K)= OR(40000000000000000000B,ISUB1(K))
      IF (KKK)    50,55,50
C LBLOC  =0
   45 CONTINUE
      WRITE(6,6) J
    6 FORMAT(1H0,5X,7HLBLOCK(,I2,5HJ = 0)
   50 CONTINUE
   55 RETURN
      END
```

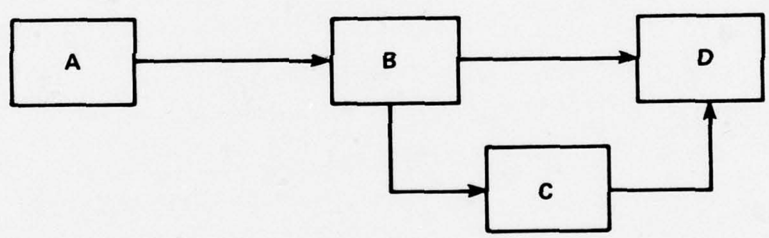
```
      SUBROUTINE BLOCUP  (J)
      DIMENSION LOAD(20)
      DIMENSION  IWORD(6)
      EQUIVALENCE (ETIME(1),IWORD(1))
      COMMON XMTBF(500),XMTTR(500),IDOWN(800),LBLOC1 (250),LBLOC2 (250)
     1,TITLE(50),ISUB1(25),ISUB2(25),SSTIME(5,25),LOST1(25),LOST2(25)
     2,LOST3(25)
      EQUIVALENCE  (LOAD,IDOWN)
      COMMON  IEQU(1500),ETIME(1500)
      COMMON STPHAS ,ENDPHA  ,NEQ,JPHASE,TP,NPH,NMI,NSMISS,NBR,NBL,NSS,
     1NSS1,ISWBL,ISWSS
C
C
      ISWBL=0
      LBLOC2(J)=AND(00000000000037777777B,LBLOC2(J))
      IHIBR=INPAKH  (LBLOC1 (J))
      ILOBR=INPAKL  (LBLOC1 (J))
      IUP=INPAKL  (LBLOC2 (J))
      ICT=IHIBR-ILOBR+1
      DO 30    K=ILOBR,IHIBR
      IF (IDOWN(K))    28,30,28
   28 ICT=ICT-1
   30 CONTINUE
      IF (ICT-IUP)    31,40,40
   31 ISWBL=1
      LBLOC2(J)=OR(40000000000000000000B,LBLOC2(J))
   40 RETURN
      END
```

APPENDIX C

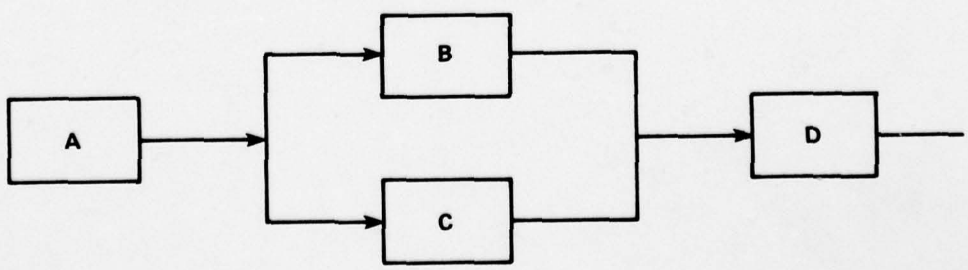
BLOCK DIAGRAM DERIVATION

The system definition must be given in terms of a reliability block diagram which is derived from a functional or schematic diagram. The initial system definition is usually given in terms of a functional diagram which describes the physical connections of all the equipment. In order to compute the reliability and the availability, this functional diagram must be transformed into a reliability block diagram. Examples of functional and block diagrams follow.

A functional diagram illustrates the interaction and relation of the components. This functional or operational (schematic) diagram reflects the actual sequence of operations--the signal path. Equipments connected in parallel are drawn in parallel; equipments connected in series are drawn in series. For example, a system might consist of items A, B, C, and D with the following schematic



The following reliability block diagram, can be derived from this functional diagram to show the effect of failures on the system.



An equipment whose failure causes the system to cease performance (mission abort) is drawn in series in the reliability block diagram, and an equipment whose failure causes mission abort only when another equipment also fails is drawn in parallel. These representations do not necessarily correspond to those in the functional diagram.

This block diagram indicates that a failure of A or D would result in mission abort, whereas if either B or C failed, the system could remain operational and the mission would not be aborted.

Once the mission scenario, block diagram, and data are known, the information can be inserted into a reliability program to obtain the required R/M quantities.

APPENDIX D
GEMJR PROGRAM DESCRIPTION

1.0 INTRODUCTION

GEMJR is an analytic model for predicting R/M. The Poisson failure process is used to develop a stochastic matrix which is solved to determine reliability and availability considering repairmen, equipment redundancies, and standbys. GEMJR was developed along the lines of the much larger GEM[2] model, in order to examine analytic R/M program operations and to compare results with those of the simulation R/M program, SIM3. GEMJR was also used to determine the feasibility of smaller R/M programs which could be used when all the power of GEM was not required.

The GEM program fills an entire CDC 6600 computer and uses its own compiler. It requires a minimum of 135,000 (octal) words of memory, 300,000 (octal) words for complicated applications. The computer program GEMJR was developed at DTNSRDC to compute reliability, availability, and other elements of maintainability. GEMJR, written in FORTRAN IV for the IBM 7090, incorporates a Poisson failure process described in Section 4. The theory and the calculations are similar to those used in GEM.

2.0 SUBROUTINE DESCRIPTIONS

The sample problem referred to in Section 3.4 will be used as a guide in describing the operation of GEMJR. GEMJR was written especially to solve a specific problem, but can be easily extended to solve other types of R/M problems.

The terminology incorporated here is a bit different than that used in the SIM3 description. In the sample problem, pictured in Figure 5, we refer to the blocks as stages, i.e., Block 1 becomes Stage 1 and so on, except that Block 4 is now divided into two stages. Previously Block 4 represented Equipments 8 and 9; now Stage 4 will consist of Equipment 8 and Stage 5 of Equipment 9. There will be a total of five stages in this problem. Otherwise, the mission is the same, consisting of three phases

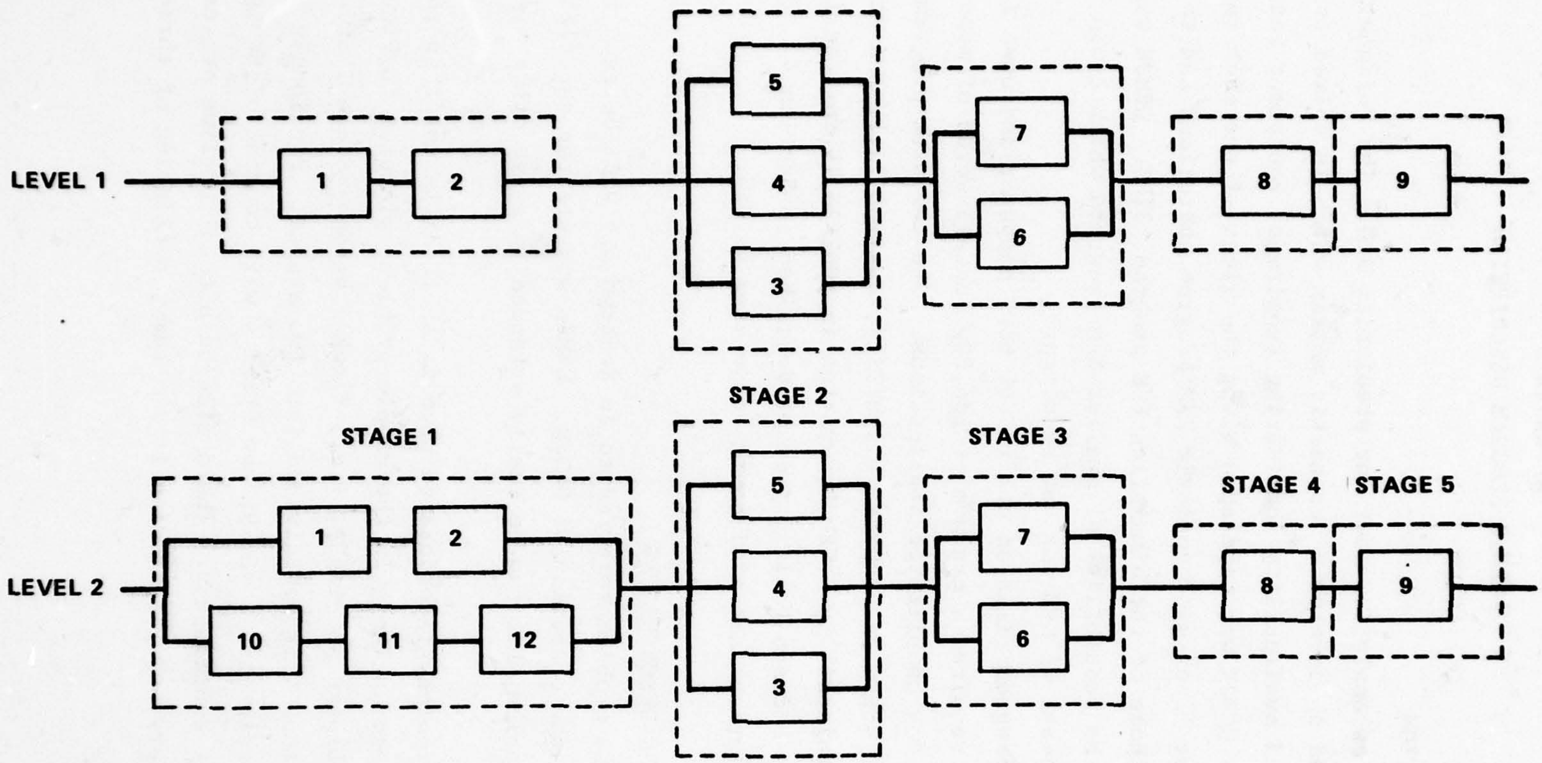


Figure 5 - Block Diagram of System A for Use with GEMJR

with two distinct operational levels. In this example we will not be concerned with the type number, and equipment number will be used only as a means of identification.

The following paragraphs describe the subroutines in the program. All required input is imbedded in these routines, and will be identified in each subroutine.

2.1 EXECUTIVE ROUTINE

This routine is the control center of the program. It specifies when and how the reliability and availability of each stage in the mission are to be computed. Most of the data required in the program are read in the executive routine.

The calculation of the reliability and availability for a specific stage is obtained by using the statement CALL HUM(J). The stage is identified by the index J which represents the number of items in that stage.

The reliability and availability calculated after each CALL HUM(J) are composite quantities. They represent values up through that stage. For example, the reliability calculated in Stage 3 represents $R_1 \times R_2 \times R_3$, where R_1 is the reliability of the first stage and so on. Since our sample problem consists of five stages, the quantities calculated during the fifth stage are the reliability and availability of the entire configuration for duration of the mission. Figure 6 indicates the subroutine calling sequence.

The following input is required in this routine:

Variable Name	Description
NST	Total number of stages in configuration
NPH	Total number of phases in mission
TOM(N), N = 1, NPH	Time for start of phase; TOM(N) represents the start of the Nth phase
TINC	Time increment, ΔT, used in iteration process (see Section 4.2)

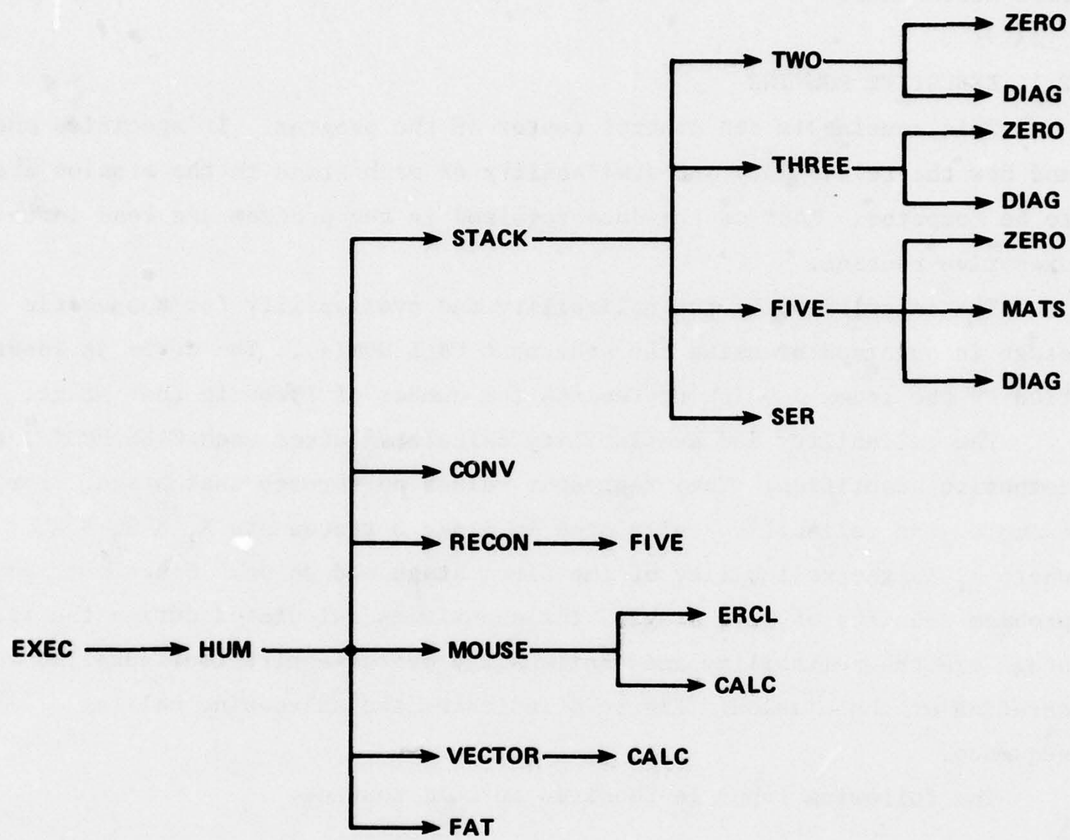


Figure 6 - GEMJR Subroutine Calling Sequence

XL, XM	Arrays containing MTBF's and MTTR's for all equipment in Stage 1
XLS, XLM	MTBF and MTTR for all other stages. (These stages do not contain more than one type of equipment.)
RBL, RAL	Arrays containing initial values of reliability and availability

2.2 SUBROUTINE HUM(JN)

JN is the number of items in the stage called.

This subroutine controls the calculations for each stage. Reliability and availability are computed up through the stage called and are printed out.

Input	Description
NIND	NIND = 0 indicates one equipment stage (Stages 4 and 5)
	NIND = 1 all other stages

The following quantities are not input but are described for clarification:

Variable	Description
A	Array containing stochastic matrix
NP	Phase number index
IP	IP = 1, reliability is calculated IP = 2, availability is calculated
NSF	Number of up-states

2.3 SUBROUTINE STACK (JN,N)

N is the number of states in the stochastic matrix. The stochastic matrix associated with the number of equipment JN is set up through calls to the appropriate subroutines.

2.4 SUBROUTINE CONV(P)

P is the state probability vector. The stochastic matrix and state probability vector associated with Stage 1 are converted from the form used in Phase 1, to one suitable for use in Phase 2.

2.5 SUBROUTINE FAT

The reliability and availability at each stage are printed out. Interval reliability is used in Phase 2 of Stage 1. To get the reliability up through Phase 2, this interval reliability must be multiplied by the reliability computed in Phase 1. To accomplish this operation the variable COT is introduced.

Variable	Description
REL(J)	For the ITth stage, the reliability at the Jth phase
RAL(J)	For the ITth stage, the availability at the Jth phase

2.6 SUBROUTINE RECON (N,P,AS)

This subroutine reconverts the stochastic matrix of Stage 1 from its altered form used during Phase 2 to its original form for use during Phase 3. This subroutine and subroutine CONV are necessary since Stage 1 undergoes a configuration change at Phase 2, requiring an alteration in the stochastic matrix.

2.7 SUBROUTINE SER

This subroutine computes reliability and availability of single equipment stages (series components).

2.8 SUBROUTINE MOUSE (N,AF,AS,NP,IP)

This subroutine sets up the infinite series in order to calculate the state probabilities. AF is the sum of the identity matrix and the first term of the infinite series. AS is the stochastic matrix A which is transformed into operational form. The first term of the infinite series is added to the identity matrix.

Variable	Description
PP	Number of terms of infinite series required to satisfy accuracy criterion
TE	Array containing truncation errors for each series calculation (printed out)

2.9 SUBROUTINE CALC (AF,AS,N,NIN,T)

The sum of NIN terms of the infinite series is calculated.

NIN is the same as PP, T is the time increment used in the iteration process, and AF is the matrix sum of the infinite series.

2.10 SUBROUTINE VECTOR (N,IP,NP,AF,AS)

This subroutine computes final stage probabilities and sums them to give the reliability and the availability.

Variable	Description
NIS	Array containing values of NIN for each series calculation (printed out)

2.11 SUBROUTINE DIAG (N)

This subroutine evaluates the diagonal elements of a stochastic matrix, given all the non-diagonal terms.

Given a stochastic matrix, A_{ij}, the sum of all the terms in the i^{th} row equals 1, i.e.,

$$\sum_{j=1}^{N} A_{ij} = 1$$

The diagonal term of the i^{th} row is A_{ii} and can be evaluated from the expression

$$A_{ii} = I - \sum_{\substack{j=1 \\ i \neq j}}^{N} A_{ij}$$

2.12 SUBROUTINE FIVE (N)

The elements of stochastic matrix for Stage 1 are set up and calculated.

2.13 SUBROUTINE THREE (N)

The elements of stochastic matrix for Stage 3 are set up and calculated.

2.14 SUBROUTINE TWO (N)

Sets up and calculates elements of stochastic matrix for Stage 2.

2.15 SUBROUTINE ZERO (N)

Assigns zero to all elements of stochastic matrix before calculating the elements.

2.16 SUBROUTINE ERCL (N,H,P,T)

The number of terms required in the infinite series to satisfy the prescribed value of 10^{-8} is calculated. At least five but not more than 25 terms are used.

H is the time increment (see TINC) and P is the number of terms required to satisfy the accuracy criterion.

2.17 SUBROUTINE MAT 5

All non-diagonal elements for the stochastic matrix of Stage 1 are calculated.

3.0 PROGRAM LISTING

```
      PROGRAM MIKE(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      COMMON/ETCH/NST
      COMMON/RST/NPH
      COMMON/MIHO/ XLS,XMS,TOM(10)
      COMMON/LOLLY/TINC,NT(10),ET(10)
      COMMON /ABC/A(50,50),XL(10),XM(10)
      COMMON/DEF/RELS(10,2),P(50,2),PZ(50,2)
      COMMON/HALT/IT,RBL(10),RAL(10) ,NIS(10,2),TE(10,2)
      COMMON/NEW/NIND,NSF,NIN
C        NIN IS THE NUMBER OF TERMS IN THE INFINITE SERIES
C        MINIMUM NUMBER OF TERMS IN SERIES IS 5
C        IF TRUNCATION ERROR IS NOT LESS THAN 1.0E-8 DETERMINE
C        HOW MANY TERMS ARE NECESSARY TO OBTAIN THIS ACCURACY
C        25 TERMS IS THE UPPER LIMIT
      TOM(1) = 0.0
      TOM(2) = 262.
C        TOM(2), TIME TO END OF PHASE 1
      TOM(3) = 430.
C        TOM(3), TIME TO END OF PHASE 2
      TOM(4) = 1200.
C        TOM(4), TIME TO END OF PHASE 3
C        NPH, NUMBER OF PHASES IN MISSION
C        NST, NUMBER OF STAGES IN RELIABILITY BLOCK DIAGRAM
      NST = 5
      NPH = 3
      TINC = .5
      DO 4 J = 1,NPH
      TOT = TOM(J + 1) - TOM(J)
      TN = TOT/TINC
      NT(J) = IFIX(TN)
      JT = NT(J)
      XNT = FLOAT(JT)
      SM = XNT*TINC
      ET(J) = TOT - SM
    4 CONTINUE
      XL(1) = 2300.
      XL(2) = 2300.
      XL(3) = 22500.
      XL(4) = 12700.
      XL(5) = 910.
      XM(1) = 4.3
      XM(2) = 4.3
      XM(3) = 2.4
      XM(4) = 2.1
      XM(5) = 4.2
      DO 1 J = 1,5
      XL(J) = 1./XL(J)
    1 XM(J) = 1./XM(J)
      DO 10J = 1,NPH
      RAL(J) = 1.0
   10 RBL(J) = 1.0
      IT = 0
      CALL HUM(5)
      XLS = 22200.
      XMS = 21.3
      CALL HUM(3)
      XLS = 19400.
      XMS = 6.4
      CALL HUM(2)
      DO 6 IY = 1,2
      XLS = 1./7000.
      CALL HUM(1)
    6 CONTINUE
      STOP
      END
```

```
      SUBROUTINE HUM(JN)
      COMMON/RST/NPH
      COMMON/HALT/IT,RBL(10),RAL(10) ,MIS(10,2),TE(10,2)
      COMMON/LOLLY/TINC,NT(10),ET(10)
      COMMON /ABC/A(50,50),XL(10),XM(10)
      COMMON/NEW/NIND,NSF,NIN
      COMMON/DEF/RELS(10,2),P(50,2),PZ(50,2)
C        RELS(I,J), I = PHASE NUMBER, J = 1 RELIABILITY, J =2 AVAILABILITY
C        P(I,J),P(I,J), I ROW NUMBER, J AS ABOVE
      DIMENSION AF(50,50),AS(50,50)
C        NSF, NUMBER OF STATES IN WHICH SYSTEM IS UP
C        INPUT FORM OF MATRIX HAS ONES SUBTRACTED FROM
C        DIAGONAL ELEMENTS
C        TRANSPOSE INPUT MATRIX AF
      CALL STACK(JN,N2)
      N = N2
      IF(NIND.EQ.0) GO TO 7
      DO 15 J = 1,N
      DO 15 I = 1,N
      K = I
      L = J
      AS(L,K) = A(I,J)
   15 CONTINUE
      DO 58 J = 1,N
      DO 58 I = 1,N
   58 A(I,J) = AS(I,J)
      DO 2 NP=1,NPH
      DO 2 IP = 1,2
      IF(NP.EQ.2.AND.IT.EQ.0.AND.IP.EQ.1) CALL CONV(PZ)
      IF(NP.EQ.3.AND.IT.EQ.0.AND.IP.EQ.1) CALL RECON(N,PZ,AS)
      N = N2
      IF(IP.EQ.1) N = NSF
      CALL MOUSE(N,AF,AS,NP,IP)
      CALL VECTOR(N,IP,NP,AF,AS)
      DO 6 J = 1,N2
    6 PZ(J,IP) = P(J,IP)
    2 CONTINUE
    7 CONTINUE
      CALL FAT
      RETURN
      END
```

```
      SUBROUTINE STACK(JN,N)
      IF(JN.EQ.2) CALL TWO(N)
      IF(JN.EQ.3) CALL THREE(N)
      IF(JN.EQ.5) CALL FIVE(N)
      IF(JN.EQ.1) CALL SER
      RETURN
      END
```

```
      SUBROUTINE FAT
      COMMON/HALT/IT,RBL(10),RAL(10) ,NIS(10,2),TE(10,2)
      COMMON/ETCH/NST
      COMMON/LOLLY/TINC,NT(10),ET(10)
      COMMON/RST/NPH
      COMMON/DEF/RELS(10,2),P(50,2),PZ(50,2)
      IT = IT + 1
      WRITE(6,1) IT
      IF(IT.EQ.NST) WRITE(6,2)
      IF(IT.EQ.NST) WRITE(6,4)   NPH
      IF(IT.EQ.1.OR.IT.EQ.2.OR.IT.EQ.3) WRITE(6,3) IT,TINC
      DO 7 J = 1,NPH
      COT = 1.0
      IF(J.EQ.2.AND.IT.EQ.1) COT = RELS(1,1)
      IF(J.EQ.3.AND.IT.EQ.1) COT = RELS(1,1)*RELS(2,1)
      RBL(J) = RBL(J)*RELS(J,1)*COT
      RAL(J) = RAL(J)*RELS(J,2)
      WRITE(6,5) J
      IF(IT.EQ.4.OR.IT.EQ.5) GO TO 10
      WRITE(6,6) RBL(J) ,NIS(J,1),TE(J,1)
      WRITE(6,8)RAL(J),NIS(J,2),TE(J,2)
      GO TO 7
   10 WRITE(6,9) RBL(J),RAL(J)
    7 CONTINUE
    1 FORMAT(1H1, 20X,18HOUTPUT UP TO STAGE,I3)
    2 FORMAT(1H0,10X ,23HTHIS IS THE LAST STAGE.)
    4 FORMAT(11X,19HTHE OUTPUT AT PHASE,I3/11X,51HREPRESENTS THE RESULTS
     1 FOR THE ENTIRE CONFIGURATION)
    3 FORMAT(1H0,9X,37HA STOCHASTIC MATRIX WAS USED IN STAGE,I3/10X,11HS
     1TEP SIZE =,F5.2,23H IN ITERATION PROCEDURE)
    5 FORMAT(1H0,20X,5HPHASE,I2,7H OUTPUT)
    6 FORMAT(1H0,20X,13HRELIABILITY =,F8.5/21X,36HNUMBER OF TERMS IN INF
     1INITE SERIES =,I3/21X,18HTRUNCATION ERROR =,E14.6)
    8 FORMAT(1H0,20X,13HAVAILABILITY=,F8.5/21X,36HNUMBER OF TERMS IN INF
     1INITE SERIES =,I3/21X,18HTRUNCATION ERROR =,E14.6)
    9 FORMAT(1H0,20X,13HRELIABILITY =,F8.5/21X,14HAVAILABILITY =,F8.5)
      RETURN
      END
```

```
      SUBROUTINE CONV(P)
      COMMON /ABC/A(50,50),XL(10),XM(10)
      COMMON/NEW/NIND,NSF,NIN
      DIMENSION              P(50,2),B(3,50),C(5)
      NSF = 8
      DO 1 I = 2,4
      C(I-1) = P(I,2)
      DO 1 J = 1,32
    1 B(I-1,J) = A(I,J)
      DO 2 I = 2,8
      P(I,2) = P(I + 3,2)
      DO 2 J = 1,32
    2 A(I,J) = A(I+3,J)
      DO 3 I = 9,11
      P(I,2) = C(I-8)
      DO 3 J = 1,32
    3 A(I,J) = B(I-8,J)
      DO 4 J = 2,4
      DO 4 I = 1,32
    4 B(J-1,I) = A(I,J)
      DO 5 J = 2,8
      DO 5 I = 1,32
    5 A(I,J) = A(I,J+3)
      DO 6 J = 9,11
      DO 6 I = 1,32
    6 A(I,J) = B(J-8,I)
      DO 7 J  = 1,32
    7 P(J,1) = P(J,2)
      RETURN
      END
```

AD-A058 707 DAVID W TAYLOR NAVAL SHIP RESEARCH AND DEVELOPMENT CE--ETC F/G 14/4
A COMPARISON OF ANALYTIC AND SIMULATION RELIABILITY AND MAINTAI--ETC(U)
AUG 78 M GRAY

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2 OF 2

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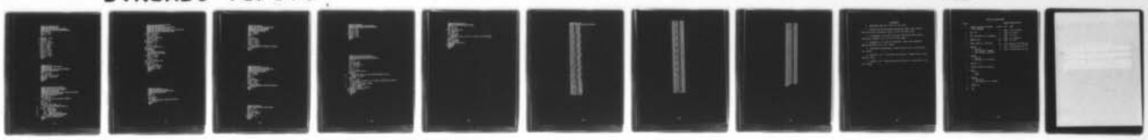

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```
      SUBROUTINE RECON(N,P,AS)
      COMMON/NEW/NIND,NSF,NIN
      COMMON /ABC/A(50,50),XL(10),XM(10)
      DIMENSION P(50,2),C(5),D(10),AS(50,50)
      CALL FIVE(N)
      DO 15 J = 1,N
      DO 15 I = 1,N
      K = I
      L = J
      AS(L,K) = A(I,J)
   15 CONTINUE
      DO 58 J = 1,N
      DO 58 I = 1,N
   58 A(I,J) = AS(I,J)
      DO 1 I = 1,3
    1 C(I) = P(I+8,2)
      DO 2 I = 2,8
    2 D(I) = P(I,2)
      DO 5 I = 2,8
    5 P(I+3,2) = D(I)
      DO 3 I = 1,3
    3 P(I+1,2) = C(I)
      DO 4 I = 1,32
    4 P(I,1) = P(I,2)
      RETURN
      END


      SUBROUTINE SER
      COMMON/HIHO/ XLS,XMS,TOM(10)
      COMMON/RST/NPH
      COMMON/DEF/RELS(10,2),P(50,2),PZ(50,2)
      COMMON/NEW/NIND,NSF,NIN
      NIND = 0
      DO 9 J = 1,NPH
      TM = TOM(J+1)
      RELS(J,1) = EXP(-XLS*TM)
      RELS(J,2) = RELS(J,1)
    9 CONTINUE
      RETURN
      END


      SUBROUTINE HOUSE(N,AF,AS,NP,IP)
      COMMON /ABC/A(50,50),XL(10),XM(10)
      COMMON/LOLLY/TINC,NT(10),ET(10)
      COMMON/NEW/NIND,NSF,NIN
      COMMON/HALT/IT,RBL(10),RAL(10) ,NIS(10,2),TE(10,2)
      DIMENSION AF(50,50),AS(50,50)
      CALL ERCL(N,TINC,PP,T)
      TE(NP,IP) = T
      NIN = IFIX(PP)
C          MATRIX AF, FIRST TWO TERMS IN SERIES
      DO 4 I = 1,N
      DO 4 J = 1,N
      PR = 0.0
      IF(I.EQ.J) PR = 1.0
      AF(I,J) = A(I,J)*TINC + PR
C          INITIALIZE OPERATIONAL MATRIX AS
    4 AS(I,J) = A(I,J)
C          AF    INPUT MATRIC
C          AF    INPUT MATRIX
C          TOT    TOTAL TIME
C          TINC    INCREMENT OF TIME
C          NIN    NUMBER OF TERMS IN SERIES
C          P    FINAL RELIABILITY VECTOR
      CALL CALC(AF,AS,N,NIN,TINC)
      RETURN
      END
```

```
      SUBROUTINE VECTOR(N,IP,NP,AF,AS)
      COMMON/NEW/NIND,NSF,NIN
      COMMON/DEF/RELS(10,2),P(50,2),PZ(50,2)
      COMMON/LOLLY/TINC,NT(10),ET(10)
      COMMON/HALT/IT,RBL(10),RAL(10) ,NIS(10,2),TE(10,2)
      DIMENSION AF(50,50),AS(50,50),PS(50)
      DO 17 J = 1,N
   17 PS(J) = PZ(J,IP)
      NTS = NT(NP)
      DO 12 KT = 1,NTS
      IF(KT.EQ.1) GO TO 25
      DO 16 J = 1,N
   16 PS(J) = P(J,IP)
   25 CONTINUE
      DO 11 I = 1,N
      Q = 0.
      DO 13 J = 1,N
   13 Q = Q + PS(J)*AF(I,J)
   11 P(I,IP) = Q
   12 CONTINUE
      EF = ET(N)
      IF(EF.EQ.0.0) GO TO 14
      CALL CALC(AF,AS,N,NIN,EF)
      DO 53 J = 1,N
   53 PS(J) = P(J,IP)
      DO 51 I = 1,N
      Q = 0.0
      DO 52 J = 1,N
   52 Q = Q + PS(J)*AF(I,J)
   51 P(I,IP) = Q
   14 REL = 0.0
      DO 3 J = 1,NSF
    3 REL = REL + P(J,IP)
      RELS(NP,IP) = REL
      NIS(NP,IP) = NIN
      RETURN
      END
```

```
      SUBROUTINE FIVE(N)
      COMMON/DEF/P(40),PZ(40),REL
      COMMON /ABC/A(50,50),XL(10),XN(10)
      COMMON/NEW/NIND,NSF,NIN
      NIND = 1
      NSF = 11
      N = 32
      CALL ZERO(N)
      CALL MATS
C        COMPUTE DIAGONAL ELEMENTS OF MATRIX A
      CALL DIAG(N)
      RETURN
      END
```

```
      SUBROUTINE THREE(N)
      COMMON /ABC/A(50,50),XL(10),XM(10)
      COMMON/HIHO/ XLS,XMS,TOM(10)
      COMMON/DEF/P(40),PZ(40),REL
      COMMON/NEW/NIND,NSF,NIN
      XLS = 1.0/XLS
      XMS = 1.0/XMS
      NIND = 1
      NSF = 2
      N = 4
      CALL ZERO(N)
      A(1,2) = 3.*XLS
      A(2,1) = XMS
      A(2,3) = 2.*XLS
      A(3,4) = XLS
      A(4,3) = 3.*XMS
C        COMPUTE DIAGONAL ELEMENTS OF MATRIX A
      CALL DIAG(N)
      RETURN
      END
```

```
      SUBROUTINE TWO(N)
      COMMON/HIHO/ XLS,XMS,TOM(10)
      COMMON /ABC/A(50,50),XL(10),XM(10)
      COMMON/DEF/P(40),PZ(40),REL
      COMMON/NEW/NIND,NSF,NIN
      XLS = 1.0/XLS
      XMS = 1.0/XMS
      NSF = 2
      NIND = 1
      N = 3
      CALL ZERO(N)
      A(1,2) = 2.*XLS
      A(2,1) = XMS
      A(2,3) = XLS
      A(3,2) = 2.*XMS
C        COMPUTE DIAGONAL ELEMENTS OF MATRIX A
      CALL DIAG(N)
      RETURN
      END
```

```
      SUBROUTINE DIAG(N)
      COMMON /ABC/A(50,50),XL(10),XM(10)
      DO 8 I = 1,N
      PA = 0.0
      DO 7 J = 1,N
      IF(I.EQ.J) GO TO 7
      PA = PA + A(I,J)
    7 CONTINUE
    8 A(I,I) = -PA
      RETURN
      END
```

```
      SUBROUTINE ZERO(N)
      COMMON /ABC/A(50,50),XL(10),XM(10)
      COMMON/DEF/RELS(10,2),P(50,2),PZ(50,2)
      DO 2 I = 1,N
      DO 2 J = 1,N
      PZ(I,1) = 0.0
      PZ(I,2) = 0.0
    2 A(I,J) = 0.0
      PZ(1,1) = 1.0
      PZ(1,2) = 1.0
      RETURN
      END
```

```
      SUBROUTINE CALC(AF,AS,N,NIN,T)
      COMMON /ABC/A(50,50),XL(10),XM(10)
      DIMENSION AF(50,50),AS(50,50),A2(50,50)
      NJ = NIN - 1
      TS = T
      DO 6 JJ = 1,NJ
      XN = FLOAT(JJ)
      TS = TS*T/(XN+1.0)
      DO 5 I = 1,N
      DO 5 J = 1,N
      Q = 0.
      DO 3 K = 1,N
      Q = Q + AS(I,K)*A(K,J)
    3 CONTINUE
      A2(I,J) = Q
C        MATRIX A2 RESULT OF MATRIX MULTIPLICATION OF AS*A,
C        WHERE AS = A**NJ
    5 CONTINUE
      DO 7 I  = 1,N
      DO 7 J = 1,N
C        SET RESULT OF MATRIX MULT. A2 EQUAL TO OPERATIONAL MATRIX AS
      AS(I,J) = A2(I,J)
C        COMPUTE NJ-1 TERM IN SERIES
      A2(I,J) = AS(I,J)*TS
C        SUM UP SERIES
    7 AF(I,J) = AF(I,J) + A2(I,J)
   40 FORMAT(3(10X,F10.5))
   12 CONTINUE
    6 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ERCL(N,H,P,T)
      COMMON /ABC/A(50,50),XL(10),XM(10)
      YA = 0.0
      DO 7 I = 1,N
      DO 7 J = 1,N
    7 YA = YA + A(I,J)**2
      YA = SQRT(YA)
      TH = YA*H
      DO 6 IP = 5,25
      P = FLOAT(IP)
      T = (TH**(P + 2.)/(P + 2.))*(1. + TH/(P + 3.))*.5*(EXP(TH)
     1  -EXP(-TH))
      IPN = IP + 1
      DO 9 J = 1,IPN
    9 T = T/FLOAT(J)
      IF(T.LE.1.E-8) GO TO 3
    6 CONTINUE
    3 CONTINUE
      RETURN
      END
```

```
SUBROUTINE MATS
COMMON /ABC/A(50,50),XL(10),XM(10)
A(1,2) = XL(1)
A(1,3) = XL(2)
A(1,5) = XL(3)
A(1,6) = XL(4)
A(1,7) = XL(5)
A(2,1) = XM(1)
A(2,4) = XL(2)
A(2,12) = XL(3)
A(2,13) = XL(4)
A(2,14) = XL(5)
A(3,1) = XM(2)
A(3,4) = XL(1)
A(3,15) = XL(3)
A(3,16) = XL(4)
A(3,17) = XL(5)
A(4,2) = XM(2)
A(4,3) = XM(1)
A(4,18) = XL(3)
A(4,19) = XL(4)
A(4,20) = XL(5)
A(5,1) = XM(3)
A(5,8) = XL(4)
A( 5,10) = XL(5)
A(5,12) = XL(1)
A(5,15) = XL(2)
A(6,1) = XM(4)
A(6,8) = XL(3)
A(6,9) = XL(5)
A(6,13) = XL(1)
A(6,16) = XL(2)
A(7,1) = XM(5)
A(7,9) = XL(4)
A( 7,10) = XL(3)
A(7,14) = XL(1)
A(7,17) = XL(2)
A(8,5) = XM(4)
A(8,6) = XM(3)
A( 8,11) = XL(5)
A(8,21) = XL(1)
A(8,22) = XL(2)
A(9,6) = XM(5)
A(9,7) = XM(4)
A( 9,11) = XL(3)
A(9,23) = XL(1)
A(9,24) = XL(2)
A(10, 5) = XM(5)
A(10, 7) = XM(3)
A(10,11) = XL(4)
A(10,25) = XL(1)
A(10,26) = XL(2)
A(11, 8) = XM(5)
A(11, 9) = XM(3)
A(11,10) = XM(4)
A(11,27) = XL(1)
A(11,28) = XL(2)
```

```
A(12, 2) = XM(3)
A(12, 5) = XM(1)
A(12,18) = XL(2)
A(12,21) = XL(4)
A(12,25) = XL(5)
A(13, 2) = XM(4)
A(13, 6) = XM(1)
A(13,19) = XL(2)
A(13,21) = XL(3)
A(13,23) = XL(5)
A(14, 2) = XM(5)
A(14, 7) = XM(1)
A(14,20) = XL(2)
A(14,23) = XL(4)
A(14,25) = XL(3)
A(15, 3) = XM(3)
A(15, 5) = XM(2)
A(15,18) = XL(1)
A(15,22) = XL(4)
A(15,26) = XL(5)
A(16, 3) = XM(4)
A(16, 6) = XM(2)
A(16,19) = XL(1)
A(16,22) = XL(3)
A(16,24) = XL(5)
A(17, 3) = XM(5)
A(17, 7) = XM(2)
A(17,20) = XL(1)
A(17,24) = XL(4)
A(17,26) = XL(3)
A(18, 4) = XM(3)
A(18,12) = XM(2)
A(18,15) = XM(1)
A(18,30) = XL(5)
A(18,31) = XL(4)
A(19, 4) = XM(4)
A(19,13) = XM(2)
A(19,16) = XM(1)
A(19,29) = XL(5)
A(19,30) = XL(3)
A(20, 4) = XM(5)
A(20,14) = XM(2)
A(20,17) = XM(1)
A(20,29) = XL(4)
A(20,30) = XL(3)
A(21, 8) = XM(1)
A(21,12) = XM(4)
A(21,13) = XM(3)
A(21,27) = XL(5)
A(21,31) = XL(2)
A(22, 8) = XM(2)
A(22,15) = XM(4)
A(22,16) = XM(3)
A(22,28) = XL(5)
A(22,31) = XL(1)
A(23, 9) = XM(1)
A(23,13) = XM(5)
```

```
A(23,14) = XM(4)
A(23,27) = XL(3)
A(23,29) = XL(2)
A(24, 9) = XM(2)
A(24,16) = XM(5)
A(24,17) = XM(4)
A(24,28) = XL(3)
A(24,29) = XL(1)
A(25,10) = XM(1)
A(25,12) = XM(5)
A(25,14) = XM(3)
A(25,28) = XL(4)
A(25,30) = XL(2)
A(26,10) = XM(2)
A(26,15) = XM(5)
A(26,17) = XM(3)
A(26,28) = XL(4)
A(26,30) = XL(1)
A(27,11) = XM(1)
A(27,21) = XM(5)
A(27,23) = XM(3)
A(27,25) = XM(4)
A(27,32) = XL(2)
A(28,11) = XM(2)
A(28,22) = XM(5)
A(28,23) = XM(3)
A(28,26) = XM(4)
A(28,32) = XL(1)
A(29,19) = XM(5)
A(29,20) = XM(4)
A(29,23) = XM(2)
A(29,24) = XM(1)
A(29,32) = XL(3)
A(30,18) = XM(5)
A(30,20) = XM(3)
A(30,25) = XM(2)
A(30,26) = XM(1)
A(30,32) = XL(4)
A(31,18) = XM(4)
A(31,19) = XM(3)
A(31,21) = XM(2)
A(31,22) = XM(1)
A(31,32) = XL(5)
A(32,27) = XM(2)
A(32,28) = XM(1)
A(32,29) = XM(3)
A(32,30) = XM(4)
A(32,31) = XM(5)
RETURN
END
```

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